Math 222 : Lie Groups and Lie Algebras

Instructor: Alison Miller **E-mail:** abmiller@math.harvard.edu
Office: Science Center 527 **Office Hours:** Mon 3-4, Fri 11-12
Meeting Time: MWF, 10-11am **Location:** Science Center 310 *(SC 229 for the first week)*
Course Assistant: TBA **Section Time and Location:** TBA
Course Website: canvas.harvard.edu/courses/9079

Course Description

This is a graduate-level first course in Lie Theory, with a focus on the representation theory of Lie groups and semisimple Lie algebras.

Tentative List of Topics

Basic definitions; relationship between Lie groups and Lie algebras. Representations of Lie groups and Lie algebras; Peter-Weyl theorem on representations of a compact Lie group. Basic theory of Lie algebras. Weights and roots. Representation theory of semisimple Lie algebras. Classification of semisimple Lie algebras.

Prerequisites

The official recommended prerequisites for this class are Math 114 (measure theory, integration, and Banach spaces), Math 123 (second-semester abstract algebra) and Math 132 (differential topology).

The background I expect students to have is the following:

Previous exposure to differential geometry: you should know what a smooth manifold is, and be comfortable working with tangent spaces and have experience handling differential forms. We'll review this material in the first week of class, and it will be used most intensively early on in the course.

Topology: you should know some basic point-set topology and be familiar with the concept of a covering space (though the latter will only be needed occasionally).

Measure theory, integration, and L^2-spaces: You should be familiar with the concept of a measure, with Lebesgue integration, and with the Hilbert space $L^2(X)$ of square-integrable functions on a measure space X. In the first half of the course we will be using these to state and prove the Peter-Weyl theorem, which is a generalization of Fourier analysis to non-abelian compact Lie groups. Previous familiarity with Fourier analysis will be helpful for this, but not necessary.

Multilinear algebra: tensor products, wedge products, symmetric products. If you haven't seen this material in a previous course, you can find it in appendix B of Fulton-Harris (see below). If necessary, I may also provide additional references at the point when we get to this material, but don't intend to spend much time reviewing it in class.

Other abstract algebra: Apart from multilinear algebra, the main necessary abstract algebra background are rigorous linear algebra (through Jordan decomposition) and the basics of groups and rings (homomorphisms, quotients, group actions). Group theory and ring theory beyond the basics are really mostly useful to motivate the analogous concepts in Lie theory. I don't expect you to have necessarily seen the representation theory of finite groups, but some familiarity with the topic is helpful for background and motivation.

Complex analysis: We'll occasionally be using the notion of a holomorphic (complex differentiable) function of several variables. It will help to have some experience with holomorphic functions in a single variable, but we won't be using any serious complex analysis; the material we need should be easy to pick up if you have the differential geometry background.

If you are concerned that you may be lacking background in some of these against, please come to me with any questions.

Textbooks and References

The main textbook for this class is *Lie groups: Beyond an Introduction, 2nd edition* by Anthony Knapp. The first edition of this book is available for free as a PDF through SpringerLink. (`http://link.springer.com/book/10.1007%2F978-1-4757-2453-0`). The main difference between the two editions is that the second edition adds a nice introductory section on closed subgroups of linear groups. We'll be covering material from the chapters 1-5, and maybe some of chapter 6.

I will also be posting PDF course notes to the Canvas website after each class.

Other recommended books are:

Representation Theory by Fulton and Harris; this is one of the standard books on the subject, and is extremely readable, with many examples. (SpringerLink URL: `http://link.springer.com/book/10.1007%2F978-1-4757-2453-0`.)

Compact Lie Groups by Sepanski: a good book at a less advanced level than Knapp; particularly useful for the beginning of the course. (SpringerLink URL: http://link.springer.com/book/10.1007/978-0-387-49158-5.)

Lectures on Lie Groups and Lie Algebras by Carter, Segal, and McDonald: the middle section of this book is my favorite overview of Lie groups, and highly recommended as supplementary reading. Not so much of a reference, as it is concise and requires you to fill in the details, but gives a good sense of the big picture.

There are a vast number of online notes available from other courses on Lie theory. Some I'd like to call to your attention are:

Kirillov's notes: http://www.math.stonybrook.edu/~kirillov/mat552/liegroups.pdf

(unofficial) Notes for a class taught by Sophie Morel at Princeton http://www-personal.umich.edu/~zhufeng/mat449.pdf (notes by Feng Zhu).

(unofficial) Notes for previous versions of this course: http://web.stanford.edu/~tonyfeng/222.pdf (taught by Schmid, notes by Tony Feng) and http://www.math.harvard.edu/~amathew/224.pdf (taught by Harris, notes by Akhil Mathew).

Homework and Grading Policies

The final grades for this class will be based 75% on weekly homework and 25% on the final paper.

Homeworks will be assigned weekly, to be turned in on Wednesdays in class (or online by 10:00 am). All homeworks will be weighted equally and the lowest two homework grades will be dropped.

The final assessment for this class will be a 5-10 page paper on a topic related to the course material.

You are encouraged to discuss the homework problems with your classmates, but you must write them up independently. You should acknowledge everyone you worked with in your homework writeups, as well as any external sources you consulted.