Physical Sciences 2 Practice
Final Exam C Saturday, December 19, 2015

Your name: ____________________________

Section TF: __________________________

Do not turn the page until you are told to begin. You will be given 3 hours to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper. Please box your final answer to each calculation.

You may use a calculator if you have brought one. You may refer to two 8.5”x11” sheets of notes, which must be in your own handwriting. Turn in your notes along with the exam itself when time is called.

This exam contains 10 sheets of paper (including this one), consisting of 8 problems.

Do not write in the following table; it will be used for grading.

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Problem 1: Multiple Choice [30 points]

For each of the following questions, circle the letter corresponding to the best answer from the options given. No partial credit will be given, except where noted. You do not need to show your work, although you are welcome to use the space on the page for scratch paper.

a) [5 points] Two identical cups are filled with water. Cup A has a block of wood floating in it, but Cup B does not. The water level on both cups is equal. Which cup (including its contents) weighs more?

A. Cup A  
B. Cup B  
C. The two cups have the same weight  
D. It is not possible to tell with the information given

![Diagram of cups A and B]

b) [5 points] The graph below shows a snapshot of the concentration of solute particles as a function of position inside a long, thin tube. The center of the tube is at $x = 0$, and $+x$ is to the right.

Fill in the blanks: at $x = 0$, the flux of solute particles is to the _________ and the concentration is ____________ in time at the instant shown.

A. right; increasing  
B. right; decreasing  
C. left; increasing  
D. left; decreasing
Problem 1: Multiple Choice (continued)

c) [5 points] A spaceship is moving in deep space, far from any stars. When must it fire its rockets? (Choose all that apply; partial credit will be given.)

   A. Constantly, just to keep moving
   B. To speed up
   C. To slow down
   D. To change direction

d) [5 points] A bacterium is moving in still water, far from any other bacteria. When must it spin its flagella? (Choose all that apply; partial credit will be given.)

   A. Constantly, just to keep moving
   B. To speed up
   C. To slow down
   D. To change direction

e) [5 points] Short answer: Karl and Lucy are discussing their answers to the above two multiple-choice questions

   Karl: Did you get the same answer for the last two multiple-choice questions?
   Lucy: No, I got different answers because momentum isn’t conserved for the bacterium in water.
   Karl: I don’t think that is right. I got different answers, too, but thought it was because the spaceship is subject to pressure drag and the bacterium is subject to viscous drag.

   You overhear Karl and Lucy and decide to interject. Explain why both of them are wrong. (3-4 sentences maximum)
Problem 1: Multiple Choice (continued)

f) [5 points] The Brownian motion of a microsphere at thermal equilibrium in an optical trap at 300 K was recorded in order to determine the effective spring constant of the trap. A graph of the resulting distribution and curve of best fit is shown below. What is the effective spring constant of the optical trap? Hint: what is the potential energy $U$ of a mass on a spring?

A. $1.56 \times 10^{-5}$ N/m  
B. $1.96 \times 10^{-6}$ N/m  
C. $3.92 \times 10^{-6}$ N/m  
D. $7.84 \times 10^{-6}$ N/m
Problem 2: Blowin’ in the wind [30 points]

A rubber balloon is filled with helium and inflated to a sphere of radius $R = 20$ cm. The balloon is closed with a string and tied to a post, and a steady breeze blows from left to right with a speed of $v = 2$ m/s.

a) [5 points] Draw a free-body diagram for the balloon in static equilibrium, clearly labeling all of the forces on it.

b) [5 points] Which of the following pictures could represent the static equilibrium position of the balloon and string? Circle your answer and explain briefly.
Problem 2: Blowin’ in the wind (continued)

c) [20 points] The rubber comprising the balloon itself has a mass of 1 gram. Assume that the helium, with a molar mass of 4 g/mol, is at the same pressure as the air outside (which averages 29 g/mol). The density of atmospheric air is roughly $\rho_{\text{air}} = 1.2 \text{ kg/m}^3$, and the balloon has a drag coefficient of $c_D = 0.2$. At equilibrium, what angle $\theta$ does the string make with the vertical?
Problem 3: U-tube.com [25 points]

A volume $V$ of water is poured into a U-shaped tube, as shown. Both ends of the tube are open to the atmosphere. The tube has a constant cross-sectional area $A$. At equilibrium, both sides of the tube have the same water level (the dashed horizontal line). However, if you displace the water slightly (say, by blowing into one end of the tube briefly), the water will slosh back and forth in the tube, undergoing harmonic oscillation.

a) [10 points] Let $y$ be the instantaneous height of the water in the left tube above the equilibrium level. Show that the total gravitational potential energy of the water $U_{grav}$ is $U_{grav}(y) = U_0 + cy^2$, where $U_0$ is the value of $U_{grav}$ when $y = 0$, and $c$ is a constant. What is the value of $c$ in terms of $A, g$, and the density of water, $\rho$?

b) [5 points] Derive an expression for the total kinetic energy of the water $K$, in terms of $V, \rho$, and $\frac{dy}{dt}$. (Hint: to a very good approximation, all of the water is moving at the same speed.)

(continued...)
Problem 3: U-tube.com (continued)

c) [10 points] If we neglect surface tension and viscosity, then the total mechanical energy of the system will be conserved. Using your answers to the previous two parts, derive a differential equation for $y(t)$ that expresses conservation of mechanical energy, and show that $y(t) = y_{\text{max}} \cos(\omega t)$ is a valid solution for the right value of $\omega$. What will be the period of the sloshing if $V = 1$ L and $A = 10$ cm$^2$?
Problem 4: Winter Vacation [20 points]

A skier of mass $M$ starts from rest at the top of a mountain, points her skis down the trail, and begins to glide. Assume that the coefficient of kinetic friction between the snow and skis is $\mu_k$.

a) [10 points] If we ignore air drag, how long will it take her to finish a straight run of distance $L$ that makes an angle $\theta$ with the horizontal? Express your answer in terms of $\mu_k$, $g$, $L$, and $\theta$. Be sure to draw a labeled free-body diagram as part of your answer.

(continued...)
Problem 4: Winter Vacation (continued)

b) [10 points] Our skier is at a sea-level ski resort, where the air has a density of 1.2 kg/m$^3$ and a viscosity of $1.8 \times 10^{-5}$ Pa s. If we include air drag, on the run described in part (a) above she will reach a terminal speed of 30 m/s.

What would be her terminal speed on a mountain near Denver (altitude = 1620 m)? Note that the viscosity of air will be roughly the same in Denver as at sea level, and you may assume $T = 0^\circ$C (273 K).
Problem 5: Microrheology [30 points]

To measure the viscosity of small volumes of biological fluids such as the cell cytoplasm, researchers use an indirect technique called microrheology. This involves adding small particles to the fluid and measuring their mean-square displacement. The figure below shows the measured two-dimensional mean-square displacement for particles of radius $R = 0.5 \pm 0.001 \mu\text{m}$ in an unknown solution at $298 \pm 0.1 \text{K}$.

![Graph showing mean-square displacement vs. time](image)

a) [15 points] Based on these experiments, how would you report the viscosity of the fluid?

(continued...)
Problem 5: Microrheology (continued)

a) (continued from previous page)

b) [15 points] You had previously measured the viscosity of a known DNA sample to be $0.011 \pm 0.001$ Pa s and a known protein sample to be $0.005 \pm 0.001$ Pa s. Could the sample you measured in part (a) be either of these samples? Explain briefly.
Problem 6: Quarter-pipe [30 points]

You push a block (\( m = 200 \text{ g} \)) against a horizontal spring (\( k = 200 \text{ N/m} \)), compressing the spring a distance \( d = 15 \text{ cm} \). The block is on a frictionless surface. Beyond the equilibrium position of the uncompressed spring is a rough surface (\( L = 60 \text{ cm} \)) that has a coefficient of friction \( \mu_k = 0.3 \). Following this surface is a frictionless curved ramp.

\[ (continued...) \]
Problem 6: Quarter-pipe (continued)

b) [10 points] Including the first trip, how many times does the block travel up the ramp before it comes to rest somewhere on the rough patch?
Problem 7: Forever Blowing Bubbles [15 points]

You can create soap bubbles filled with air (density $\rho_{\text{air}}$) by dipping a plastic ring (radius $r$) in an aqueous soap solution (density $\rho_{\text{soap}}$, liquid-gas surface tension $\gamma$) and blowing into the ring.

With what speed $v$ do you need to blow to keep a bubble of radius $R$ inflated? Assume that air inside the bubble is not moving, which means the streamline stops at the ring.
Problem 8: Quality Time (20 points)

While on vacation at a beach resort, you observe a father spinning around in a circle while holding his daughter by her arms, as shown. He is spinning around a vertical axis that runs from his head to his feet. The daughter is hanging at an angle $\theta$ from this axis.

Using the photograph and your PS2 skills, estimate the angular speed of the father. (We are looking for a numerical answer, with calculations that support your result.) For simplicity, you may treat the daughter as a point mass located a distance $R$ from the axis of rotation. Please state clearly any assumptions you are making, and be sure to include a free-body diagram.