Recognize Geometric Sequences  The first person generates 5 emails. If each of these people sends the email to 5 more people, 25 emails are generated. If each of the 25 people sends 5 emails, 125 emails are generated. The sequence of emails generated, 1, 5, 25, 125, ... is an example of a geometric sequence.

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a nonzero constant $r$ called the common ratio. The common ratio can be found by dividing any term by its previous term.

\[
\frac{10}{5} = 2
\]

\[
5, 10, 20, 40, ...
\]
Determine whether each sequence is arithmetic, geometric, or neither. Explain.

a. 256, 128, 64, 32, ...

Find the ratios of consecutive terms.

\[
\frac{128}{256} = \frac{1}{2} \quad \frac{64}{128} = \frac{1}{2} \quad \frac{32}{64} = \frac{1}{2}
\]

Since the ratios are constant, the sequence is geometric. The common ratio is \(\frac{1}{2}\).

b. 4, 9, 12, 18, ...

Find the ratios of consecutive terms.

\[
\frac{9}{4} = \frac{9}{4} \quad \frac{12}{9} = \frac{1}{3} \quad \frac{18}{12} = \frac{1}{2}
\]

The ratios are not constant, so the sequence is not geometric.

Find the differences of consecutive terms.

\[
12 - 9 = 3 \quad 18 - 12 = 6
\]

There is no common difference, so the sequence is not arithmetic. Thus, the sequence is neither geometric nor arithmetic.
<table>
<thead>
<tr>
<th>Guided Practice</th>
<th>1A–1C. See margin.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1A.</strong> 1, 3, 9, 27, ...</td>
<td><strong>1B.</strong> −20, −15, −10, −5, ...</td>
</tr>
<tr>
<td>geometric common ratio 3</td>
<td>arithmetic common difference 5</td>
</tr>
</tbody>
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- **1A** is geometric with a common ratio of 3.
- **1B** is arithmetic with a common difference of 5.
- **1C** is neither; no common difference or ratio.
Example 2  Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence.

a. 1, −4, 16, −64, ...

**Step 1** Find the common ratio.

\[
\begin{align*}
\frac{-4}{1} &= -4 \\
\frac{16}{-4} &= -4 \\
\frac{-64}{16} &= -4
\end{align*}
\]

**Step 2** Multiply each term by the common ratio to find the next three terms.

\[
\begin{align*}
-64 \times (-4) &= 256 \\
256 \times (-4) &= -1024 \\
-1024 \times (-4) &= 4096
\end{align*}
\]

The next three terms are 256, −1024, and 4096.

Find common ratio \( \frac{-4}{1} = \boxed{-4} \)

\[
\frac{-64}{16} = -4
\]
b. 9, 3, 1, \frac{1}{3} ...

**Step 1** Find the common ratio.

\[
\begin{align*}
9 \quad & \quad 3 \quad & \quad 1 \quad & \quad \frac{1}{3} \\
\frac{3}{9} & = \frac{1}{3} \quad & \quad \frac{1}{3} & = \frac{1}{3} \quad & \quad \frac{1}{3} \quad & = \frac{1}{3} \\
\end{align*}
\]

The value of \( r \) is \( \frac{1}{3} \).

**Step 2** Multiply each term by the common ratio to find the next three terms.

\[
\begin{align*}
\frac{1}{3} \quad & \quad \frac{1}{9} \quad & \quad \frac{1}{27} \quad & \quad \frac{1}{81} \\
\times \frac{1}{3} \quad & \quad \times \frac{1}{3} \quad & \quad \times \frac{1}{3} \\
\end{align*}
\]

The next three terms are \( \frac{1}{9} \), \( \frac{1}{27} \), and \( \frac{1}{81} \).

\[
\frac{3}{q} = \frac{1}{3}
\]
Guided Practice

2A. \( -3, 15, -75, 375, \ldots \)

\[
\frac{15}{-3} = -5 \\
-1875, \\
9375, \\
-46875
\]

2B. \( 24, 36, 54, 81, \ldots \)

\[
\frac{36}{24} = 1.5 \times \left( \frac{3}{2} \right) \\
121.5, 182.25, 273.375
\]
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2-8 (even)
and
14-24 (even)