Law of Total Probability

If $E_1, \ldots, E_n$ are disjoint,

$$P(A) = \sum_{i=1}^n P(A|E_i) P(E_i).$$

Example: Roll two dice, what is the probability that the sum is 7?

- $S = \{1, \ldots, 6\}$, uniform dist.
- $E_{11} = \{1, 1\} \subset \{1, 2, \ldots, 6\}$
- $P(E_{11}) = \frac{1}{36}$
- $P(A|E_{11}) = \frac{1}{6}$

Independence

Events $A$ and $B$ are independent if

$$P(A \cap B) = P(A) P(B).$$

Law of Multiplication

$$P(A \cap B) = P(A) P(B|A).$$

Law of Union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Law of Total Probability

$$P(A) = \sum_{i=1}^n P(A|E_i) P(E_i).$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$