AB CALCULUS (0 to 100)
Stuff you MUST know Cold

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined OR at endpoints

local minimum:

$\frac{dy}{dx}$ goes (–,0,+) or (–,und,+) or $\frac{d^2y}{dx^2} > 0$

local maximum:

$\frac{dy}{dx}$ goes (+,0,–) or (+,und,–) or $\frac{d^2y}{dx^2} < 0$

point of inflection: concavity changes

$\frac{d^2y}{dx^2}$ goes from (+,0,–), (–,0,+), (+,und,–), or (–,und,+)

Basic Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin u\right) = \cos u \bullet u'$$

$$\frac{d}{dx}\left(\cos u\right) = -\sin u \bullet u'$$

$$\frac{d}{dx}\left(\tan u\right) = \sec^2 u \bullet u'$$

$$\frac{d}{dx}\left(\cot u\right) = -\csc^2 u \bullet u'$$

$$\frac{d}{dx}\left(\sec u\right) = \sec u \tan u \bullet u'$$

$$\frac{d}{dx}\left(\csc u\right) = -\csc u \cot u \bullet u'$$

$$\frac{d}{dx}\left(\ln u\right) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left(e^u\right) = e^u \frac{du}{dx}$$

More Derivatives

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}} \bullet u'$$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2} \bullet u'$$

$$\frac{d}{dx}\left(\cot^{-1} u\right) = \frac{-1}{1+u^2} \bullet u'$$

$$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{|u|\sqrt{u^2-1}} \bullet u'$$

$$\frac{d}{dx}\left(\csc^{-1} u\right) = \frac{-1}{|u|\sqrt{u^2-1}} \bullet u'$$

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a \bullet u'$$

$$\frac{d}{dx}\left(\log_a u\right) = \frac{1}{u \ln a} \bullet u'$$

Differentiation Rules

Chain Rule

$$\frac{d}{dx}\left[f(u)\right] = f'(u)\frac{du}{dx} \quad OR \quad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad OR \quad uv' + vu'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad OR \quad \frac{vu' - uv'}{v^2}$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$

Corollary to FunThmCalculus

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)$$

Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and k is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = k$.

Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, AND the first derivative exists on the interval (a, b), then there is at least one number $x = c$ in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

* Rolle's Theorem: f '(c) = 0.

Area Formulas

Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$
Square	$A = s^2$
Rectangle	$A = lw$
Triangle	$A = \frac{1}{2}bh$

Numerical Methods for Integration

Trapezoidal Rule

$$\int_a^b f(x)dx = \tfrac{1}{2}\tfrac{b-a}{n}\left[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$$

RRAM (Right-hand Rect. Approx.)
LRAM (Left-hand Rect. Approx)
MRAM (Midpt. Rect. Approx)

Theorem of the Mean Value
i.e. AVERAGE VALUE

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

Solids of Revolution and friends

Disk Method

$$V = \pi\int_{x=a}^{x=b}\left[R(x)\right]^2 dx \qquad \text{(about x-axis)}$$

Washer Method

$$V = \pi\int_a^b\left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right)dx$$

(about x-axis)

Cross Sections

$$V = \int_a^b Area(x)\, dx \qquad \perp to\ x-axis$$

Position, Velocity, and Acceleration

velocity = $\frac{d}{dt}$(position)

acceleration = $\frac{d}{dt}$(velocity)

speed = $|v|$

displacement = $\int_{t_o}^{t_f} v\, dt$

distance = $\int_{\text{initial time}}^{\text{final time}} |v|\, dt$

average velocity =

$$\frac{\Delta s}{\Delta t} = \frac{\text{final position} - \text{initial position}}{\text{total time}}$$

average acceleration =

$$\frac{\Delta v}{\Delta t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{total time}}$$

Asymptotes

Example: $y = \frac{x-a}{x-b}$

Vertical Asymptote: x = b
* goes with infinite limit as x → b

Horizontal Asymptote: y = 1
* goes with limits at infinity (3 rules)

Related Rates

Variables changing with respect to TIME! Use implicit diff.

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left[r^2 \frac{dh}{dt} + h2r\frac{dr}{dt}\right]$$

Integration

Area, Sum, Accumulation → Integrate
Integral of Rate = Total or Net Change

Differentiation

Slope, Instantaneous Rate of Change → Differentiate
Derivative = Slope of Tangent Line

Differentiability

No cusps, corners, vertical tangents, or discontinuity

Continuous Function at a point

$$\lim_{x\to a^-} = \lim_{x\to a^+}$$

$$\lim_{x\to a} = f(a)$$

Values of Trigonometric Functions for Common Angles

Θ	sin θ	cos θ	tan θ
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	"∞"
Π	0	–1	0

Basic Trig Integrals

1. $\int \sec x \tan x \, dx = \sec x + C$
2. $\int \cos x \, dx = \sin x + C$
3. $\int \sec^2 x \, dx = \tan x + C$
4. $\int \sin x \, dx = -\cos x + C$
5. $\int \csc^2 x \, dx = -\cot x + C$
6. $\int \csc x \cot x \, dx = -\csc x + C$
7. $\int \tan x \, dx = -\ln|\cos x| + C$
8. $\int \cot x \, dx = \ln|\sin x| + C$
9. $\int \sec x \, dx = \ln|\sec x + \tan x| + C$
10. $\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

More Integrals

1. $\int du = u + C$
2. $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$
5. $\int e^u du = e^u + C$
4. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
6. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
7. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec \frac{|u|}{a} + C$

Inverse Trig Functions:

arc sin (0) = sin^{-1}(0)

arc cos(½) = cos^{-1}(½)

arc tan (1) = tan^{-1}(1)

FTC I (another version)

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

FTC II (easy version)

$$y = \int_a^x f(t)\,dt$$

$$y' = f(x)$$

Definition of a Derivative

"h is the same as delta x"

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

Separation of Variables

$$\frac{dy}{dt} = ky$$

$$y = Ce^{kt}$$

Slope Fields

Graph of tiny slopes of a given differential equation, representing all solutions to that differential equation.

Trig Identities

Pythagorean

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Reciprocal

$\sec x = \frac{1}{\cos x}$ *or* $\cos x \sec x = 1$

$\csc x = \frac{1}{\sin x}$ *or* $\sin x \csc x = 1$

Odd-Even

sin(–x) = – sin x (odd)
cos(–x) = cos x (even)

Optimization / Extreme Value Thm.

1. Write function in terms of one variable.
2. Find the first derivative and set it equal to zero.
3. Check the endpoints if necessary.

Optimum means either maximum (highest value) or minimum (lowest value).

Implicit Differentiation

$$2x + y^2 = y$$

$$2 + 2yy' = y'$$

$$2yy' - y' = -2$$

$$y'(2y - 1) = -2$$

$$y' = \frac{-2}{2y-1}$$