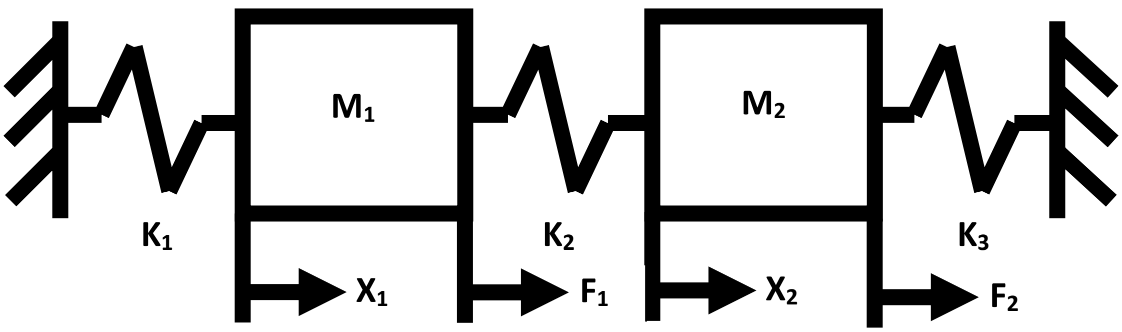
# EMA 540 Exam

One note sheet and a graphing calculator are allowed.

**Problem #1:** For the system shown below:




$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (K_1 + K_2) & -K_2 \\ -K_2 & (K_2 + K_3) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$M_1 = M_2 = 0.5$[kg]　　$K_1 = K_3 = 1$[N/m]　　$K_2 = 2$[N/m]

**A)** The system becomes **SDOF** in $X_1$ when $X_2$ is fixed at zero displacement (so that it acts like a fixed boundary condition). Determine the free vibration response of $X_1$ when it is released with initial displacement of $X_1 = 1$[m] and $\dot{X}_1 = 1$[m/s] initial velocity. (Hint: The response of the mass is known to be of the form $x_1 = Re\left[(a + ib)e^{i\omega t}\right]$, so solve for a and b.)

**B)** Write the uncoupled equations of motion for the undamped, forced, **MDOF** system in the time domain. Then, solve for the steady-state response If the applied forces are harmonic forces, $F_1=2\sin(t)$ and $F_2=\cos(2t)$, the mass normalized modes are $\Phi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ $\Phi_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ and the natural frequencies are $\omega_1=1.41$ and $\omega_2=3.16$.

**C)** Suppose that a dashpot is connected between masses 1 and 2. If a damping matrix is added where $C = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}$, what would the modal damping coefficient $\zeta_2$ be for the second mode?

**D)** Write out the response vector $X(\omega) = [X_1(\omega)\ X_2(\omega)]^T$ for the steady-state response of the MDOF, damped system, using [C] from part (c), when the following harmonic forces $F_1=2\sin(\omega *t)$ and $F_2=\cos(\omega *t)$ are applied. **Solve it two ways:** once directly using the M, C, and K matrices, and once using the modal equations and the light damping approximation. Write out all matrices and expressions that would be input into Matlab; you do not need to evaluate any matrix products or matrix inverses.

E) There should also be some concept question on here about FFTs, for example how to set up the problem to solve in Matlab, or similar to P5 in HW#5, or something like that.