Short-Time Fourier Transform:

Objective: Analysis of dynamic signals that are “not-stationary” (i.e., evolve over time).

Example: 3 pure tones (20, 70 and 110 Hz), “non overlapping” in time.

Signal 1 (non-stationary): Time domain representation

Question: What is frequency content? Do not know
Finding: Frequencies are obscure, So, let’s use FFT

Signal 1 (non-stationary): Frequency domain representation, \( F_{res} = 0.5 \) Hz.

Q: When does each frequency occur? Do not know
Finding: Description in time is obscure
Signal 2 (Stationary): Time domain representation

This signal also has 20, 70 and 110 Hz, but all happening at the same time.

Finding: Frequencies are obscure. Then, use FFT.

Signal 2 (Stationary): Frequency domain representation, $F_{res} = 0.5 \, \text{Hz}$.

Findings, This signal has the same spectrum as above. We do not know when each frequency occurs.

**Question:** Can we see time and frequency information simultaneously?

**Solution:** Calculate the time-frequency distribution, using “Short-time FFT”

Use a sliding window in time and calculate the spectrum at each time step

This results in a 2-D plot called “Spectrogram”
Signal 2 (Stationary): Time-Frequency distribution (Fres=0.5 Hz, Tres=2 sec).

Finding: We can see the persistence of frequencies over the whole duration as expected.

Signal 1 (Non-stationary): Time-Frequency distribution (Fres=0.5 Hz, Tres=2 sec).

Finding: We can see that the frequency varies over time (i.e. can get information over time and frequency)

Problem: There is overlap in frequencies which is incorrect.
Now we choose a time resolution of 0.06 sec ($F_{\text{res}} = 16$ Hz).

Signal 1 (Non-stationary): Time-Frequency distribution

![Time-Frequency distribution](image)

Finding: Can see clear separation of the zones where each frequency occurs.

Frequency resolutions has decayed.

Check Frequency domain representation (Signal 1).

![Frequency domain representation](image)

Finding: A coarse frequency resolution can be seen

What to do now??
Signal 1 (non-stationary): Frequency domain representation, $F_{res} = 2$ Hz.

Finding: We need to find a compromise between time and frequency resolution.

Signal 1 (Non-stationary): Time-Frequency distribution ($F_{res}$=2 Hz, $T_{res}$=0.5 sec).

Finding: We need to find a compromise between time and frequency resolution.
More applications:


Example: Decaying Chirp.
Example: Seismocardiographic signal (Heart emitted vibrations).