**Solving Compound Inequalities**

1. **Inequalities Containing and** When considered together, two inequalities such as \( h \geq 52 \) and \( h \leq 72 \) form a compound inequality. A compound inequality containing \( \text{and} \) is only true if both inequalities are true. Its graph is where the graphs of the two inequalities overlap. This is called the intersection of the two graphs.

   The intersection can be found by graphing each inequality and then determining where the graphs intersect:
   
   \[
   \begin{align*}
   x &\geq 3 \\
   x &< 7 \\
   x &\geq 3 \text{ and } x < 7 \\
   3 &\leq x < 7
   \end{align*}
   \]

   \[x \geq 3 \text{ and } x < 7\] can be read as \( x \) is greater than or equal to 3 and less than 7 or \( x \) is between 3 and 7 including 3.

2. **Guided Practice**
   Solve each compound inequality. Then graph the solution set.
   
   1A. \( y \leq -3 \geq -11 \) and \( y < -3 \leq -8 \)
   1B. \( 6 \leq r + 7 < 10 \)

3. **Inequalities Containing or** Another type of compound inequality contains the word \( \text{or} \). A compound inequality containing \( \text{or} \) is true if at least one of the inequalities is true. Its graph is the union of the graphs of two inequalities.

   \[
   \begin{align*}
   x &> 2 \\
   x &< -1 \\
   x &> 2 \text{ or } x < -1
   \end{align*}
   \]

   When solving problems involving inequalities, **within** is meant to be inclusive, so use \( \geq \) or \( \leq \). Between is meant to be exclusive, so use \( < \) or \( > \).

4. **Real-World Example**
   **Write and Graph a Compound Inequality**

   **SOUND** The human ear can only detect sounds between the frequencies 20 Hertz and 20,000 Hertz. Write and graph a compound inequality that describes the frequency of sounds humans cannot hear.

   The problem states that humans can hear the frequencies between 20 Hz and 20,000 Hz. We are asked to find the frequencies humans cannot hear.

   \[f < 20 \text{ and } f > 20,000\]

   \[f \leq 20 \text{ or } f \geq 20,000\]

   \[f = 20,000\]

   \[f \neq 20,000\]

   **Example:**
   Solve and graph an intersection.

   \[2 \leq x - 3 < 4\]

   First, express \( 2 \leq x - 3 < 4 \) using **and**. Then solve each inequality.

   \[2 \leq x - 3 \text{ and } x - 3 < 4\]

   \[-1 \leq x \text{ and } x < 7\]

   Simplify.

   \[1 \leq x \text{ and } x < 7\]

   Now graph the solution set.

   \[\text{Graph } 1 \leq x \text{ or } x < 7\]

   \[\text{Graph } x < 7\]

   \[\text{Find the intersection of the graphs.}\]
Example 3: Solve and Graph a Union.

Solve \(-2m + 7 \leq 13\) or \(5m + 12 > 37\). Then graph the solution set.

\[-2m + 7 \leq 13\]
\[
-2m + 7 \leq 13 - 7
\]
\[
-2m \leq 6
\]
\[
-m \leq 3
\]
\[
m \geq -3
\]

\[
5m + 12 > 37
\]
\[
5m + 12 > 37 - 12
\]
\[
5m > 25
\]
\[
m > 5
\]

Notice that the graph of \(m \geq -3\) contains every point in the graph of \(m > 5\). So, the union is the graph of \(m \geq -3\). The solution set is \([m | m \geq -3]\).

Guided Practice
Solve each compound inequality. Then graph the solution set.

3A. \(a + 1 < 4\) or \(a - 1 \geq 3\)

3B. \(r \leq 9\) or \(2 + 4r < 10\)

Practice and Problem Solving

Examples 1, 3: Solve each compound inequality. Then graph the solution set.

1. \(4 \leq p - 8\) and \(p - 14 \leq 2\)
2. \(r + 6 < -8\) or \(-r - 3 > -10\)
3. \(4 + 7 \leq 31\) or \(n > 5\)
4. \(2 \leq g + 4 < 7\)

Example 2

5. **Sense-Making** The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike’s tires have 24 pounds per square inch, what is the recommended range of air that should be put into the tires?

Practice and Problem Solving

Examples 1, 3: Solve each compound inequality. Then graph the solution set.

6. \(-6 \leq -6\) and \(-6 < 4 \geq 2\)
7. \(n + 2 \leq -5\) and \(n + 6 \geq -6\)
8. \(-y + 1 \geq 7\) or \(-y + 3 < -1\)
9. \(t + 14 \geq 15\) or \(-t - 9 \geq -10\)
10. \(-3 \leq -4 < 18\)
11. \(-3 \leq -6 < 14\)
12. \(-5 \leq -4 < 6\) and \(-7 < -11 < 32\)
13. \(-12 \leq -4 < 13\)
14. \(-5 \leq -5 < 10 < 6 < 14\)
15. \(-4 \leq -4 < 3 < 13\)
16. \(-4 < -4 < 3 < 13\)
17. \(-5 \leq -5 < 3 < 13\)
18. \(-5 < -5 < 3 < 13\)
19. \(-5 < -5 < 3 < 13\)
20. \(-5 < -5 < 3 < 13\)
21. \(-5 < -5 < 3 < 13\)
22. \(-5 < -5 < 3 < 13\)
23. \(-5 < -5 < 3 < 13\)
24. \(3b + 2 < 6\) or \(-6 < 9\)
25. \(-2a + 3 > -1 \geq 3a - 10\)
26. \(10m - 7 < 17\) or \(-6m > 36\)
27. \(5m - 1 < -16\) or \(-3m < 8\)
28. **Coupon** Juanita has a coupon for 10% off on any digital camera at a local electronics store. She is looking at digital cameras that range in price from $100 to $220.
   a. How much are the cameras after the coupon is used?
   b. If the tax amount is 6.5%, how much should Juanita expect to spend?

Define a variable, write an inequality, and solve each problem. Then check your solution.

29. Eight less than a number is no more than 14 and no less than 5.
30. The sum of 3 times a number and 4 is between -8 and 10.
31. The product of -5 and a number is greater than 35 or less than 10.
32. One half a number is greater than 0 and less than or equal to 1.
33. **Snakes** Most snakes live where the temperature ranges from 75°F to 90°F, inclusive. Write an inequality for temperatures where snakes will not thrive.
34. **Fundraising** Yuma is selling gift cards to raise money for a class trip. He can earn prizes depending on how many cards he sells. So far, he has sold 34 cards. How many more does he need to sell to earn a prize in category 4?
35. **Turtles** Atlantic sea turtle eggs that incubate below 22°C or above 33°C rarely hatch. Write the temperature requirements in two ways:
   a. as a pair of simple inequalities, and as a compound inequality.
36. **Structure** The Triangle Inequality Theorem states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.
   a. Write and solve three inequalities to express the relationships among the measures of the sides of the triangle shown at the right.
   b. What are four possible lengths for the third side of the triangle?
   c. Write a compound inequality for the possible values of x.
33. HURRICANES  The Saffir-Simpson Hurricane Scale rates hurricanes on a scale from 1 to 5 based on their wind speed.

- Write a compound inequality for the wind speeds of a category 3 and a category 4 hurricane.
- What is the intersection of the two graphs of the inequalities you found in part a?

38. MULTIPLE REPRESENTATIONS  In this problem, you will investigate measurements. The absolute error of a measurement is equal to one half the unit of the measure. The relative error of a measure is the ratio of the absolute error to the expected measure.

- Tabular: Copy and complete the table.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Absolute Error</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.3 cm</td>
<td>0.2 cm</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>1.83 cm</td>
<td>0.05 cm</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>1.0 cm</td>
<td>0.1 cm</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>2.02 cm</td>
<td>0.05 cm</td>
<td>0.05 cm</td>
</tr>
</tbody>
</table>

- Analytical: You measured a length of 12.8 centimeters. Compute the absolute error and then write the range of possible measures.
- Logical: To what precision would you have to measure a length in centimeters to have an absolute error of less than 0.05 centimeter?
- Analytical: To find the relative error of an area or volume calculation, add the relative errors of each linear measure. If the measures of the sides of a rectangular box are 6.5 centimeters, 7.2 centimeters, and 10.25 centimeters, what is the relative error of the volume of the box?

46. GRIDDED RESPONSE  What is the next term in the sequence?

47. After paying a $15 membership fee, members of a video club can rent movies for $2. Nonmembers can rent movies for $4. What is the least number of movies which must be rented for it to be less expensive for members?

A 9  C 7  B 8  D 6

Spiral Review

48. BABYSITTING  Marilyn earns $150 per month delivering newspapers plus $7 an hour babysitting. If she wants to earn at least $300 this month, how many hours will she have to babysit? [Lesson 5-5]

49. MAGAZINES  Carlos has earned more than $260 selling magazine subscriptions. Each subscription was sold for $12. How many did Carlos sell? [Lesson 5-5]

50. PUNCH  Raquel is mixing lemon-lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice? [Lesson 5-4]

51. Solve each proportion. If necessary, round to the nearest hundredth. [Lesson 2-9]

52. Determine whether each relation is a function. Explain. [Lesson 1-7]

53. Evaluate each expression. [Lesson 1-2]

54. Solve each equation. [Lesson 9-3]

H.O.T. Problems

39. ERROR ANALYSIS  Chloe and Jonas are solving $3 < 2x - 5 < 7$. Is either of them correct? Explain your reasoning.

Chloe: $3 < 2x - 5 < 7$

Jonas: $3 < 2x - 5 < 7$

40. RESIST PERSISTENCE  Solve each inequality for $x$. Assume $x$ is constant and $a > 0$.

- $-3 < ax + 1 < 5$
- $-2 < x + a < 1$ or $-ax > 8$

41. OPEN ENDED  Create an example of a compound inequality containing or that has infinitely many solutions.

42. CHALLENGE  Determine whether the following statement is always, sometimes, or never true. Explain. The graph of a compound inequality that involves an or statement is bounded on the left and right by two values of $x$.

43. WRITING IN MATH  Give an example of a compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union?