a) Which of the following statements **must always** be true for an isolated system? *(Assume no friction/drag in all cases.)* **Circle ALL that apply:** partial credit will be given.

A) Its center of mass must be at rest.

B) The velocity of its center of mass must be constant.

C) If you add \( m_1 \dot{v}_1 + m_2 \dot{v}_2 + m_3 \dot{v}_3 + \ldots \) for all objects in the system, the sum will be zero.

D) If you add \( m_1 \dot{v}_1 + m_2 \dot{v}_2 + m_3 \dot{v}_3 + \ldots \) for all objects in the system, the sum will be constant.

E) Its total momentum cannot change over time.

b) A SPACEX rocket is lifting off with a velocity \( \vec{v}_{RG} \) with respect to the ground. You are driving your car toward the launch pad with a velocity \( \vec{v}_{CG} \). What is the correct expression for the velocity of the rocket with respect to your car? *(Assume no friction/drag in all cases.)*

A) \( \vec{v}_{RG} - \vec{v}_{CG} \)

B) \( \vec{v}_{CG} - \vec{v}_{RG} \)

C) \( -\vec{v}_{RG} - \vec{v}_{CG} \)

D) \( \vec{v}_{RG} + \vec{v}_{CG} \)

c) Which of the following is true for a projectile shot at an angle greater than 0° but less than 90°? Assume the projectile is shot over level ground (on earth) at a speed of 5 m/s. Neglect air drag. **Circle ALL that apply:** partial credit will be given.

A) The moment the projectile begins its trajectory, its momentum (magnitude) will begin to decrease.

B) As it flies, the x-component (horizontal component) of its velocity remains constant.

C) As it flies, the y-component (vertical component) of its velocity remains constant.

D) The larger the initial angle, the more time it will spend in the air before landing back on the ground.

E) The larger the angle it is initially shot at the farther it will travel horizontally before landing back on the ground.
d) The graph at right represents the $x$-position of a ball being rolled back and forth between two children. Which of the following is the correct graph of the $x$-velocity $v_x$, as a function of time?

![Graphs of $v_x$ vs. $t$]

- (A) $v_x$ (m/s) with $v_x = 1.5$ for $4 \leq t \leq 12$, $v_x = 1$ for $12 \leq t \leq 24$, and $v_x = -1.5$ for $24 \leq t \leq 28$
- (B) $v_x$ (m/s) with $v_x$ increasing linearly from $0$ at $t = 4$ to $1.5$ at $t = 28$
- (C) $v_x$ (m/s) with $v_x$ increasing from $0$ at $t = 4$, reaching a maximum at $t = 8$, and then decreasing back to $0$ at $t = 28$
- (D) $v_x$ (m/s) with $v_x$ constant at $1.5$ for $4 \leq t \leq 28$

e) A flying squirrel is curled up in a ball falling at her terminal velocity. To slow down, she opens her squirrel "wings." At the instant that she opens her "wings," the drag force on her is:

A) Smaller in magnitude and opposite in direction to her weight.

B) Equal in magnitude and in the same direction as her weight.

C) Equal in magnitude and opposite in direction to her weight.

D) Greater in magnitude and in the same direction as her weight.

E) Greater in magnitude and opposite in direction to her weight.
d) [5 pts] A person pulls on a block by applying a force $F_{app}$, and the block remains at rest. The force vectors in the diagram correctly show the directions, but not necessarily the magnitudes, of the various forces on the block. Which of the following relations among the force magnitudes $F_{app}, F_g, F_N$, and $f$ must be true? Here $f$ is frictional force and $F_N$ is the normal force.

A) $F_{app} = f$ and $F_N = F_g$
B) $F_{app} = f$ and $F_N > F_g$
C) $F_{app} > f$ and $F_N < F_g$
D) $F_{app} > f$ and $F_N = F_g$
E) None of the above choices are correct.
Bouncy Ball

A 100 g rubber ball is thrown horizontally with a speed of 5 m/s toward a wall. It is initially traveling to the left. It rebounds with no loss of speed (see figure).

The collision force $F_x$ is shown in the top graph below. 

**Assume that gravity is not acting on the ball** (i.e. there is no gravitational force acting on the ball).

a) [5 pts] What is the magnitude and direction of the maximum force $F_{\max}$, as denoted on the graph?

$$F_{\max} = \frac{m \omega_x - m \omega_{\Delta t}}{\Delta t} = \frac{5 - (-5)}{10 \times 10^{-3}} m/s^2 = 100 \text{ N}$$

b) [5 pts] Draw an acceleration-versus-time graph for the collision on the middle set of axes (starting from t=0). Make your graph align vertically with the force graph, and provide an appropriate numerical scale on the vertical axis.

c) [5 pts] Draw a velocity-versus-time graph for the collision on the bottom set of axes (starting from t=0). Make your graph align vertically with the acceleration graph, and provide an appropriate numerical scale on the vertical axis.

$$\text{Initial} \quad \vec{v}_i = -5 \frac{m}{s}$$

$$\text{Final} \quad \vec{v}_f = 5 \frac{m}{s}$$

$$F_x \text{ N}$$

10 ms

1000

0

$\text{Final}$

$\text{Initial}$

$$\vec{v}_f = 5 \frac{m}{s}$$

$$\vec{v}_i = -5 \frac{m}{s}$$
A couple is figure skating in a pair (assume no friction). He weighs 60 kg, and she weighs 40 kg. He’s originally skating due east, carrying her in his arms, and skating at 2 m/s. Then she leaps from his arms. After she jumps, she is traveling (with respect to the ice) at 4 m/s southeast (i.e. 45° south of east).

What is his speed and direction after she jumps? (Express his direction in terms of the number of degrees from the east.)

\[
\vec{P}_i = \vec{P}_f
\]

\[
100 \text{ kg} (2, 0) \text{ m/s} = 40 \text{ kg} (4 \cos 45°, 4 \sin 45°) + 60 \text{ kg} (V_x, V_y)
\]

\[
(200 - 160 \cos 45°, 160 \sin 45°) \text{ kg m/s} \cdot \frac{1}{60 \text{ kg}} = (V_x, V_y)
\]

\[
(8.7, 11.3) \text{ m/s} = (V_x, V_y)
\]

\[
(1.45, 1.9) \text{ m/s} = (V_x, V_y)
\]

\[
\text{Speed} = \sqrt{1.45^2 + 1.9^2} = 2.4
\]

\[
\theta = \tan^{-1} \left( \frac{1.9}{1.45} \right) = 57° \text{ N of E}
\]

continued...
b) One second after she leaves his arms, what is the position of their center of mass (i.e. the center of mass of the system that includes both him and her)? Set the origin at the location where she leapt from his arms, the positive x-axis pointing east, and the positive y-axis pointing north.

The system of the two of them is isolated, so their center of mass continued at 2 m/s east even after she leaps from his arms. So after 1 second, the center of mass is 2 meters east of the origin.

$$\vec{r}_{\text{com}} = (2\text{m}, 0\text{m})$$
Swinging a stone

A child ties a stone of mass \( m = 0.5 \) kg to a nylon string and swings it around in a vertical circle, as shown. The string will break if the tension force exceeds \( F_{\text{max}} = 400 \) N. Assume the child swings the stone at constant speed \( v \).

Calculate the maximum value of \( v \) such that the string will not break at the bottom of the circle. Show all your work clearly (FANCLAN would be a good idea here).

\[ F_T \]

\[ F_{\text{grw}} \]

\[ m \cdot g = F_T - F_{\text{grw}} \]

\[ \text{Centripetal accel: } \quad q_g = \frac{V^2}{L} \]

\[ m \cdot \frac{V^2}{L} = F_T - mg \]

Solve for \( F_T = \frac{mV^2}{L} + mg \)

If \( F_T = F_{\text{max}} \), solve for \( V^2 = L \left( \frac{F_T}{m} - g \right) \)

or \( V = \sqrt{L \left( \frac{F_{\text{max}}}{m} - g \right)} = \sqrt{(1m) \left( \frac{400N}{0.5 \text{kg}} - 9.8 \frac{m}{s^2} \right)} \)

\[ = 28.1 \frac{m}{s} \]

\[ \text{(28.3 } \frac{m}{s} \text{ if ignore growth)} \]
A mountain goat jumps down the side of a mountain to escape from a predator. She lands on the side of the mountain and starts sliding down with initial speed $v_0$ at an angle $\theta$ from the horizontal. Although the mountainside is rough, you may model it simply as a flat plane with a large coefficient of kinetic friction.

Find an expression for the distance $D$ that the goat will slide before coming to rest. Express your answer in terms of the speed $v_0$, the angle $\theta$, the goat's mass $m$, the kinetic friction coefficient $\mu_k$ between the goat and the mountain, and any relevant physical constants. FANCLAN would be a good idea.

\[ X: \quad -F_k + F_g \sin \theta = m a_x \]
\[ Y: \quad F_N - F_g \cos \theta = mg \quad \theta \]
\[ a_x = \frac{1}{m} \left[ F_g \sin \theta - F_k \right] = \frac{1}{m} \left[ m g \sin \theta - \mu_k F_N \right] \]
\[ a_x = g \sin \theta - \mu_k g \cos \theta \]

Now kinematics:
\[ v_x^2 = v_{0x}^2 + 2a \Delta x \]
\[ \Delta x = \frac{v_{0x}^2}{2a_x} = \frac{v_0^2}{2g[\mu_k \cos \theta - \sin \theta]} \]

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