Name: ______________________________

EMA 540 – Mid-Term Exam – M.S. Allen
Fall 2020

Honor Pledge: On my honor, I pledge that this exam represents my own work, and that I have neither given nor received inappropriate aid in the preparation of this exam.

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Signature

You are allowed one sheets of notes for this exam (both sides). Calculators are allowed, but you must show all of your work to receive credit.

Formulas:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Free Response for SDOF:
The general solution to an underdamped SDOF system

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n{}^2 x = 0$$

is:

$$x(t) = \mathrm{Re}\left(Ae^{-\zeta\omega_n t}e^{\mathrm{i}\omega_d t}\right)$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and A is a complex constant.

Forced Steady-State Response:

$$f(t) = \mathrm{Re}\left(Fe^{\mathrm{i}\omega t}\right) \rightarrow x(t) = \mathrm{Re}\left(Xe^{\mathrm{i}\omega t}\right)$$

Fourier Series

$$Q(t) = \frac{1}{2}F_0 + \mathrm{Re}\left[\sum_{n=1}^{\infty} F_n \exp(in\omega_1 t)\right]$$

$$Q(t) = \frac{1}{2}\sum_{n=-\infty}^{\infty} F_n \exp(in\omega_1 t),\ F_{-n} = F_n^*$$

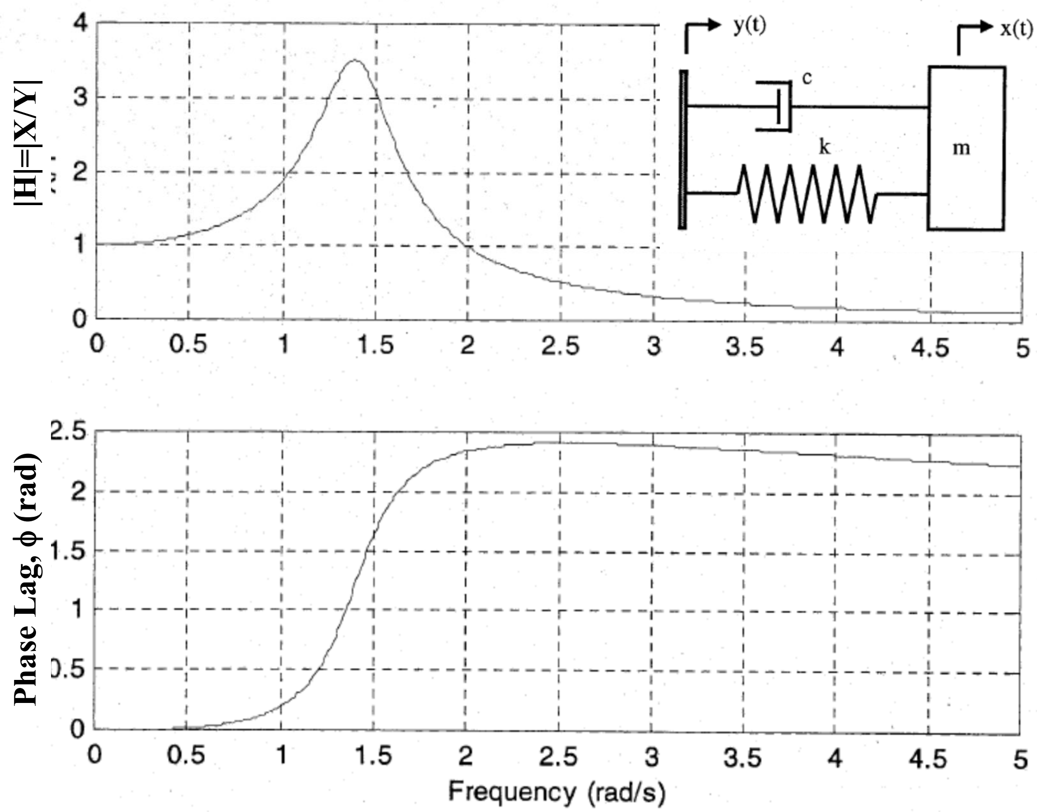
$$F_k = \frac{2}{T}\int_{-T/2}^{T/2} Q(t)\exp(-ik\omega_1 t)\,dt$$

$$F_n = \frac{2}{N}F_{DFT}$$

Problem #1 (50 pts)
The magnitude and phase characteristics of a base-excited linear system with harmonic excitation are shown below. The top graph shows the *magnitude* of the displacement $x(t)$ over the magnitude of the input displacement $y(t)$, while the bottom graph shows the *phase lag*, ϕ, in radians as a function of the frequency in rad/s (i.e. the transfer function between input y and output x is $H=X/Y=|H|e^{-i\phi}$). Answer the following questions, using values estimated from the curves below. Mark the points on the graph from which you've estimated your values, and show what values you obtained.

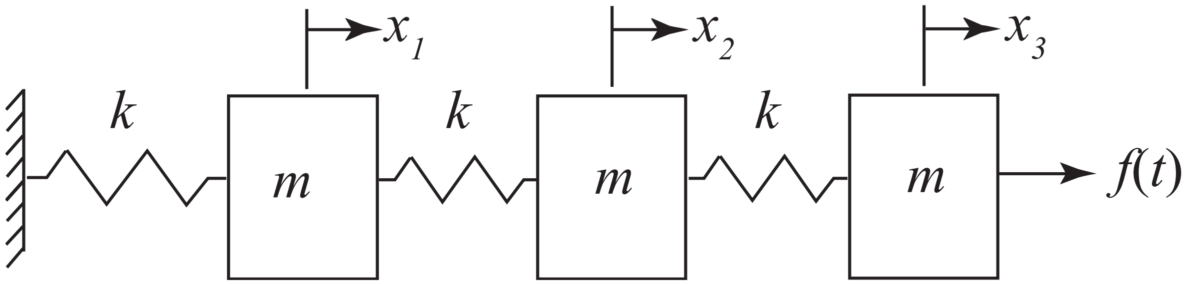
(a) If the input to the system is $y(t) = 4+\sin(t)+3\cos(2t)$, give an expression for the steady-state displacement output, $x(t)$. (Hint: if you are struggling to understand, convert y(t) to a sum of complex exponentials in polar form.)

(b) If the input to the system is $y(t) = 1.5\sin(\omega t)$, and the motion of the mass is measured to be $x(t) = -4.5\cos(\omega t)$, find the frequency, ω.



Problem #2 (50 pts)

Consider the mass-spring system shown.



The equations of motion for this system are given below, in non-dimensional form so you do not need to worry about any units for k and m.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

(a) The first two nondimensional natural frequencies are $\omega_1 = 0.45, \omega_2 = 1.2$ and the three <u>mass normalized</u> eigenvectors are given below. What is the third natural frequency? Work this out using orthogonality and without solving the eigenvalue problem.

$$\boldsymbol{\phi}_1 = \begin{bmatrix} 0.33 \\ 0.59 \\ 0.74 \end{bmatrix} \quad \boldsymbol{\phi}_2 = \begin{bmatrix} 0.74 \\ 0.33 \\ -0.59 \end{bmatrix} \quad \boldsymbol{\phi}_3 = \begin{bmatrix} -0.59 \\ 0.74 \\ -0.33 \end{bmatrix}$$

(b) If the damping matrix is that given below, what is the damping ratio of the first mode (using the light damping approximation)?

$$[C] = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.04 & -0.1 \\ 0 & -0.1 & 0.02 \end{bmatrix}$$

(c) Suppose the initial conditions given below are applied to the system and there is no applied forcing, i.e. {f(t)}=0 and the damping is zero, i.e. [C]=[0]. Write all of the equations needed to find the transient response of the system. Set up all of the equations that you would need so that this is ready to type into Matlab, but you do not need to multiply anything out.

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \\ \dot{x}_3(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$