#1 $x_1(t) = Re[(a+ib)e^{i\omega_n t}]$ $\omega_n = \sqrt{\frac{3}{0.5}} = 2.45$ rad/s

$x_1(0) = 1 = Re[(a+ib)e^{0}] \rightarrow 1 = Re[a+ib] = a$

$a = 1$

$\dot{x}_1(0) = 1 = Re[(a+ib)\, i\omega_n e^{i\omega(0)}] \rightarrow 1 = Re[(a+ib)\,\omega_n i] = Re[ai-b]\omega_n$

$\rightarrow \frac{-1}{\omega_n} = b$

#2 $\underbrace{\underline{\Phi}^T \underline{M} \underline{\Phi}}_{\underline{I}} \ddot{\underline{\eta}} + \underbrace{\underline{\Phi}^T \underline{K} \underline{\Phi}}_{[\omega_n^2]} \underline{\eta} = \underline{\Phi}^T \underline{F}$ since $\underline{x} = \underline{\Phi}\underline{\eta}$

$F_1 = 2Re[-ie^{it}]$

$F_2 = Re[e^{i2t}]$

$\underline{F} = \begin{bmatrix} 2Re[-ie^{it}] \\ Re[e^{i2t}] \end{bmatrix}$

$\underline{\Phi} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \underline{\Phi}^T$

$\underline{\Phi}^T \underline{F} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2Re[-ie^{it}] \\ Re[e^{i2t}] \end{bmatrix} = \begin{bmatrix} (2Re[-ie^{it}] + Re[e^{i2t}]) \\ ((2Re[-ie^{it}] - Re[e^{i2t}]) \end{bmatrix}$

$\rightarrow \boxed{\ddot{\underline{\eta}} + \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \end{bmatrix} \underline{\eta} = \begin{bmatrix} (2Re[-ie^{it}] + Re[e^{i2t}]) \\ (2Re[-ie^{it}] - Re[e^{i2t}]) \end{bmatrix}}$

#3

(a) proportional $K[I] + B[\omega_n^2] = \underline{\Phi}^T \underline{C} \underline{\Phi}$

for node 2

$1 + 0.1\omega_2^2 = \zeta_2 \cdot 2 \cdot \omega_2$

$1 + 1 = 2\omega_2 \zeta_2$

$\frac{2}{2\omega_2} = \frac{1}{\omega_2} = 0.316 = \zeta_2$

(b) $\underline{\underline{\Phi}}^T \underline{\underline{C}} \underline{\underline{\Phi}} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0.4 \end{bmatrix}$

$$\zeta_2 = \frac{0.4}{2\omega_2} = 0.0633$$

#4

$$\underline{\underline{M}}\,\ddot{\underline{x}} + \underline{\underline{C}}\,\dot{\underline{x}} + \underline{\underline{K}}\,\underline{x} = \underline{F}$$

$$X(\omega) = \sum_{i=1}^{N} \frac{\underline{\Phi}_j \underline{\Phi}_j^T \underline{F}}{\omega_j^2 - \omega^2 + i 2 \zeta_j \omega_j} = H(\omega)\,\underline{F}$$