

Ray Theory: Amplitude and Phase

February 5, 2020

But first: Exercise 4 of Chapter 4

Exerc. 4

$$V(z) = a + bz$$

Show that ray paths are circular.

Without loss of generality, define z so $a=0$ and

$$V(z) = bz$$

$$U = \frac{1}{bz}$$

$$p = \text{constant}$$

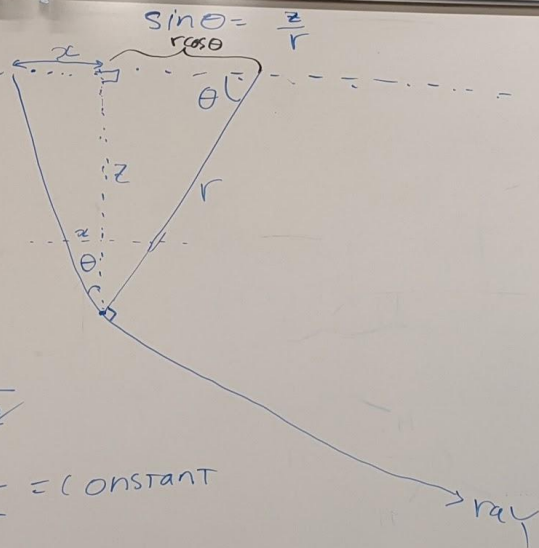
$$= \frac{\sin \theta}{V(z)}$$

$$= \frac{\sin \theta}{bz}$$

$$= \frac{z}{rbz}$$

$$p = \frac{1}{br} = \text{constant}$$

$$r = \frac{1}{bp} = \text{constant}$$



$$\begin{aligned} \mathcal{X}(p) &= p \int_0^z \frac{dz}{\sqrt{U^2 - p^2}} \\ &= p \int_0^z \frac{dz}{\sqrt{\frac{1}{b^2 z^2} - p^2}} \\ &= p \int_0^z \frac{z dz}{\sqrt{\frac{1}{b^2} - p^2 z^2}} \\ &= p \int_0^z \frac{z dz}{p \sqrt{\frac{1}{b^2 p^2} - z^2}} \\ &= \int_0^z \frac{z dz}{\sqrt{\frac{1}{b^2 p^2} - z^2}} \\ &= \int_0^z z \left(\frac{1}{b^2 p^2} - z^2 \right)^{-1/2} dz \end{aligned}$$

Exercise 4 of Chapter 4 - continued

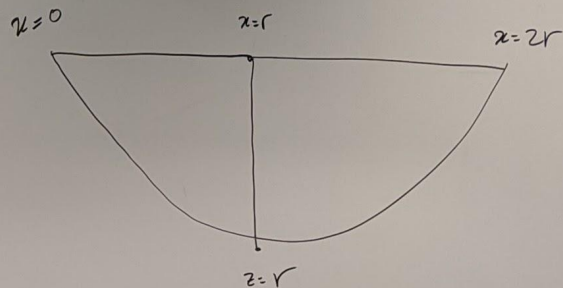
$$-\left(\frac{1}{b^2 p^2} - z^2\right)^{1/2} \Bigg|_0^z$$

$$-\sqrt{\frac{1}{b^2 p^2} - z^2} - \left(-\sqrt{\frac{1}{b^2 p^2}}\right)$$

$$\sqrt{\frac{1}{b^2 p^2}} - \sqrt{\frac{1}{b^2 p^2} - z^2}$$

$$= r - \sqrt{r^2 - z^2}$$

$$= r - r \cos \theta$$



Amplitudes of seismic waves

- Decrease due to geometrical spreading
- Decrease due to anelasticity (intrinsic attenuation)
- Decrease or increase due to interaction with discontinuities
- Small scale heterogeneity have an overall effect of decreasing phase amplitudes due to scattering

Energy density $\rightarrow \tilde{E} = \tilde{E}_k + \tilde{E}_w$

\tilde{E}_k kinetic

\tilde{E}_w (potential energy strain energy)

$$\frac{1}{2}mv^2$$

$$\tilde{E}_k = \frac{1}{2}\rho \dot{u}^2 \leftarrow \text{particle motions } \frac{\partial u}{\partial t} \text{ where } u = \text{displacement.}$$

$$\tilde{E}_w = \frac{1}{2} \tau_{ij} \epsilon_{ij}$$