1. \( f(x) = 6(2x^2 - 12x + 3) \).
   Since \( 6 = \gcd(12, 72, 18) \) the content is 6.
   
   \[ f(x) = 2 \cdot 3 \cdot (2x^2 - 12x + 3) \]
   The factors 2 and 3 are irreducible in \( \mathbb{Z} \) and in \( \mathbb{Z}[x] \). The factor \( 2x^2 - 12x + 3 \) is also irreducible: if it were to split into 2 linear factors it would have its roots in \( \mathbb{Q} \). But the usual quadratic formula gives roots
   
   \[ \frac{12 \pm \sqrt{144 - 24}}{4} = \frac{12 \pm \sqrt{120}}{4} = \frac{6 \pm \sqrt{30}}{2} \]
   which are not in \( \mathbb{Q} \).

2. Any \( x \in D \) by UFD assumption is a product \( q_1 \cdots q_s \) of irreducibles. But each \( q_i \) has the form \( q_i = pu_i \) with \( u_i \) a unit. Hence any \( x \) has the form \( p^s \cdot u \) where \( u = u_1 \cdots u_s \) is a unit.

   If \( I \subset D \) is an ideal, let \( n \) be the minimal value for which \( I \) has an element of the type \( x = p^n \cdot u \). Then \( I \) also has \( x \cdot u^{-1} = p^n \) and so \( (p^n) \subset I \). If \( y \in I \setminus (p^n) \) then \( y = p^l \cdot v \) with \( v \) a unit and \( l < n \) (otherwise \( y \in (p^n) \)). But this contradicts minimality of \( n \). So \( I = (p^n) \).

3. If (i) failed then any two irreducibles in \( R \) would be associates. By previous problem, any ideal would be principal, and this contradicts the assumption (\( R \) is not a UFD).
ideal would be principal, and this contradicts the assumption (R is not a PID).

(ii) Let $I$ be non-principal and $x^0 \in I$. Then $x= p_1 \cdots p_s$, a product of irreducibles. Let $h$ be the largest product of primes $p_1 \cdots p_s$ such that $h$ divides all nonzero elements of $I$. Then $I= hJ$ and $J$ is still non-principal. In addition, no irreducible divides all elements of $J$ (it would have to divide $x$ and thus would be one of the $p_i$). Take a maximal $M$ which contains $J$. If $M = (z)$ then $z$ is not a unit so $z = q_1 \cdots q_t$ with $q_i$ irreducible. Then $q_1$ divides all elements of $M$, hence all elements of $J$. Contradiction.

\[ x^4 + 4y^4 = x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2 = (x^2 + 2y^2)^2 - (2xy)^2 = (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy) \]

If $x^2 + 2y^2 + 2xy$ was reducible then substitution of $y=1$ would give reducible polynomial to, but $x^2 + 2x + 2$ is irreducible in $\mathbb{Q}[x]$ by Eisenstein Criterion at $p=2$.

Consider a surjective ring homomorphism

\[ \psi : D[x] \to D \]

\[ f(x) \mapsto f(0) \]

By the First Isomorphism Theorem

\[ D = \text{Im}(\psi) \cong D[x]/\text{Ker}(\psi). \]

Ker$(\psi)$ is given by polynomials with zero constant term, i.e. Ker$(\psi) = (x)$. 
So \( D[x]/(x) \cong D \).

Since \( D \) is a domain, \( (x) \) must be prime. By assumption of the problem it will also be maximal. But then \( D[x]/(x) \) is a field, as required.

\( \circ \) If \( a \), \( b \), and any \( f \) such that \( a f, b f \) must satisfy \( f \).

If \( a = p_1^{a_1} \ldots p_s^{a_s} \) with \( p_1, p_2, \ldots, p_s \) non-associate (\( i \neq j \)) and irreducible and similarly \( b = q_1^{b_1} \ldots q_t^{b_t} \), then adding trivial factors \( p_i^{0} \) or \( q_i^{0} \) we can assume that \( s = t \) and each \( p_i \) is an associate of \( q_i \).

Then \( \text{lcm}(a, b) = \prod_{i=1}^{s} p_i^{\max(a_i, b_i)} \)

This is similar to \( \text{gcd}(a, b) = \prod_{i=1}^{s} p_i^{\min(a_i, b_i)} \)