Birthday Paradox

In a room of \( n \) people, what's the probability that two people have the same birthday?

- \( D \) = set of possible birthdays
- Distribution of birthdays is uniform over \( D \times D \times \ldots \times D = D^n \)
- \( E \) = set of sequences no two are the same.

\[
P(E) = \frac{|E|}{|D|^n} = \frac{1}{10} \left( \frac{9}{10} \right) \left( \frac{8}{10} \right) \ldots \left( \frac{10-n}{10} \right) = 1 \cdot \left( 1 - \frac{1}{10} \right) \left( 1 - \frac{2}{10} \right) \ldots \left( 1 - \frac{n-1}{10} \right)
\]

\[
\Rightarrow e^{-\frac{n(n-1)}{200}} \quad \text{for } n = 1, 2, \ldots
\]

\[
\Rightarrow e^{-\frac{n(n-1)}{200}} \quad \text{if } \frac{n(n-1)}{2} \ge 10, \text{ or } n \ge \sqrt{200} \approx 14.14
\]