Physical Sciences 2: Offline Homework 0: Sept 3 - Sept 12
Vectors, Position, and Velocity
Due Thursday, Sept. 12, at 9:00 am

This assignment must be turned in by 9:00 am on Thursday, Sept. 12. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to the mailbox marked with your section TF’s name in the row of mailboxes outside of Sci Ctr 108.
You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

Grading for the offline homework is based on completion: any honest effort at answering a question will earn full credit, whether the answer is correct or incorrect. Answers left blank, or showing only minimal effort, will earn no credit.

1. Vector Mistakes (2 pts). Find the mistakes (if any) in the following statements involving vectors.
   a) \( \vec{u} + \vec{v} = 1 \)
   b) \( v_x = -5 \text{ m/s} \) and \( v_y = v_z = 0 \), so \( \vec{v} = -5 \text{ m/s} \)

   As discussed in lecture, the electrocardiogram (often called an EKG, based on its German spelling) is a device that measures the electrical activity of the heart. A skilled interpreter can use an EKG to diagnose many cardiac problems; we will focus on just one aspect of the EKG.
   The following figure shows a diagram of the interior of the heart. The most powerful pump in the heart is the left ventricle, which pumps oxygenated blood through the body. The second-most powerful pump is the right ventricle, which pumps deoxygenated blood through the lungs. (Note that the designations right and left refer to the patient’s right and left, while these illustrations are shown from the perspective of the doctor.)

![Inside the Heart](http://www.livescience.com/images/human_heart_graphic_03.jpg)
When the heart muscle contracts, an electrical signal known as a depolarization wave travels through the nerves and muscle. An EKG can detect this electrical signal through electrodes that are attached to various locations on the patient’s body. One important piece of information that can be gleaned from an EKG is the mean electrical axis of the heart. The mean electrical axis is a vector that points, roughly, in the direction of the primary muscular contraction of the ventricles of the heart. The EKG shows the components of this vector, and from these components a doctor can determine the direction of the mean electrical axis. (Yes, doctors need to know about vectors and components!)

The left illustration below shows the “electrical system” (the primary nerves) of the heart. The electrical signal for each heartbeat starts at the sinoatrial (SA) node, proceeds to the atrioventricular (AV) node, and then travels through the left and right bundle branches to the ventricles. Once the electrical signal reaches the ventricles, the signal propagates through the muscle tissue from the interior to the exterior of the heart, as shown by the small arrows in the right illustration below. Each small arrow represents the path of the depolarization wave through the muscle tissue.

The mean electrical axis of the heart is essentially the vector sum of all of the small “depolarization vectors” in the right illustration. As you can see, the sum of all of those vectors is a vector that points roughly along the axis of the heart; given the orientation of the heart in a patient’s body, this vector usually points “down and to the left” (the patient’s left).

A standard EKG has 12 leads (and provides 12 signals at once), but we will focus on only two of these leads, known as lead I and lead aVF. Lead I provides the horizontal component of the mean electrical axis, while lead aVF provides the vertical component of the mean electrical axis. The aVF lead is oriented so that positive values point down; a normal patient will usually have positive values for both lead I and lead aVF. The diagram at right shows the coordinate system used in the EKG, with the axes for lead I and lead aVF indicated. The heart is shown in its typical orientation along with a typical mean electrical axis. (The shaded area of the diagram shows the “normal range” for the mean electrical axis.) The direction of the mean electrical axis is usually reported in degrees, measured clockwise starting from lead I (as you can see from the degree markings around the circle).
Here’s an example of a typical EKG. There are six (6) graphs, representing six electrical signals: I, II, III, aVR, aVL, and aVF; focus only on the graphs marked **I** and **aVF**. The vertical axis is millivolts; the horizontal axis is time. The scale is indicated by the rectangular pulse at the far left, which has a height of 1 millivolt (mV). (1 mV corresponds to 2 large “squares” of the graph paper.)

Examine the very sharp peaks in the graph. For lead I, we see peaks with a height of about 1.0 mV. Thus, the horizontal component of the vector is +1.0. For lead aVF, we see peaks with a height of about 0.6 mV. Thus, the vertical component of the vector is +0.6. From these components, we find that the angle of the mean electrical axis is about +30°:

```
I

+1.0
```

```
aVF

+0.6
```

Sometimes the main peak on an EKG (known as the *QRS complex*) will not consist of a single peak, but will instead go “up and down” like the example shown at right. In such cases, you should consider the reading to be the height of the “up” peak minus the height of the “down” peak. Thus, in the example at right, the “up” peak on lead I is about 0.5 mV, and the “down” peak is about 1.0 mV, so the total is \((0.5 - 1.0) = -0.5\) mV. In this example, the horizontal component of the mean electrical axis is *negative*. Can you see that the mean electrical axis in this example must lie outside of the “normal” region as shown on the diagram at the bottom of page 2?
Now it’s your turn:
a) Determine the mean electrical axis for these EKG’s. Be sure to show a clear diagram of the components and the overall axis, and show all calculations explicitly. Do any of the axes lie outside of the normal range?

**Patient A:**

![EKG diagram for Patient A]

**Patient B:**

![EKG diagram for Patient B]
Patient C:

b) The mean electrical axis of the heart does not always lie along the center of the heart; usually, it lies somewhat to the left of the center of the heart. (That is, although the center of the heart lies at an angle of ~50°, the mean electrical axis usually is closer to ~30°.) Given that the left ventricle is larger and stronger than the right ventricle, explain why the electrical axis of the heart normally lies to the left. (Hint: the mean electrical axis is the sum of many “depolarization vectors.”) A clear diagram will be essential to your answer!

c) A 23-year-old female patient had the EKG shown at right. She was diagnosed with right ventricular hypertrophy, a condition in which her right ventricle was abnormally enlarged. In what way does the EKG support this diagnosis? Provide a quantitative calculation and a qualitative explanation.

d) You may have noticed that there are no axes on these EKG’s. The scale of these graphs is set by the rectangular calibration pulse at the far left of each EKG trace. This pulse has a height of 1 mV and a width of 0.20 seconds. Given this information, what is the heart rate (in beats per minute) of the female patient whose EKG is shown at right?
3. **Position and Velocity: Throwing a Baseball (4 pts).**

Major-league baseball pitchers can throw fastballs above ~90 miles per hour (~40 m/s). Since the ball is held in the pitcher’s hand until it is released, the pitcher’s hand must be moving at the same speed as the ball. These high speeds place tremendous stress on various parts of the body. To improve pitching and reduce injuries, pitchers can have their pitches analyzed by high-speed video cameras. Let’s analyze some parts of a pitching video from the American Sports Medicine Institute, asmi.org. (You can watch the video here: [http://www.youtube.com/watch?v=5OWd8VHiVKQ&list=PL2tFtB7YE1Sd2Czqidx2BL_2YO2QpA2Ld](http://www.youtube.com/watch?v=5OWd8VHiVKQ&list=PL2tFtB7YE1Sd2Czqidx2BL_2YO2QpA2Ld))

The video is slowed down by a factor of 18, so one second of “real time” takes 18 seconds in the video.

We analyzed this video using *Logger Pro* computer software (the same software that you will be using in the lab). At time intervals of 1/54 of a second, we plotted the location of three important points: the **baseball**, the pitcher’s **right shoulder**, and the center of the pitcher’s **waist**. These data are shown on the graph below, superimposed on the first frame of the video. From this graph, you can determine the position and velocity of these points throughout the pitch. (Note that this analysis considers only two dimensions of motion; the third dimension can be obtained by using other camera angles.)

![Graph showing position and velocity data for a pitching video.](image)
Let’s analyze a few short segments of the motion of this pitch.

a) Consider the path of the baseball as the pitcher brings his arm forward, as shown here; you should be able to find the corresponding segment on the video. Assuming that the motion of the ball is only in two dimensions, what is the speed of the baseball during this segment? Is the speed roughly constant? Is the velocity roughly constant during this segment? Explain your answers.

b) Now examine the ball after it leaves his hand (here the symbols each represent only $\frac{1}{450}$ of a second). What is the speed of the ball? What are the components of the velocity of the ball? What is the direction in which the ball is released from his hand? How does the speed of this pitch compare with that of a major-league fastball (~95 mph)?
c) Now let’s consider the **relative motion** of the ball, the shoulder, and the waist during another segment of the pitch (shown at the bottom of this page). In this segment, the ball is moving up, while the shoulder and waist are both moving forward (to the right). Again, assuming that all the motion is confined to this two-dimensional plane, determine, for this segment:

i) the average velocity of the ball, the shoulder, and the waist (you’ll need components here).

ii) the average velocity of the ball *relative to* the shoulder.

iii) the average velocity of the shoulder *relative to* the waist.

d) Two doctors, Abby and Beth, are discussing the motion of the shoulder during the segment of the pitch shown below. Abby says that the shoulder is clearly rising during this segment of the pitch, while Beth says that it is moving strictly in a horizontal direction. Can they both be correct? Given that the stress on various parts of the body depends on their relative positions, whose observation is more relevant? Explain briefly.

---

**Please note:**
- The entire graph is scaled to the size of the video: an object at the center of the video will be at the center of this graph.
- The video is slowed down from "real time" by a factor of 18. (Thus, one second of real time is 18 seconds in the video.)
- Each symbol on this graph represents 1/54 of a second in "real time," or 1/3 of a second in "video time."