Corr Sci 212
1. Induction
2. Pitfalls of Induction
3. Strong Induction

Announcements
- Homework 1, out
- Advice: List of things you don’t have to prove

Induction
- Proof. P(0) - Base Case
- Proof that for all n, if P(n) then P(n+1)
- Inductive step

Theorem. If P(0) is true, and P(n) implies P(n+1) for all non-negative integers n, then P(n) is true for all n (aka Induction works).

Proof. Use contradiction. Let n be the
smallest integer for which P(n) is false (Osmum contradiction principle).
n+1, because P(n) is true.
P(n+1) is true, but P(n+1) implies
P(n), which contradicts the fact that
P(n) is false.

(If n=0, P(0): P(-1))

Theorem. Let F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}
(Fibonacci number) n ≥ 2.

F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8
n=4, F_9 = F_8 + F_7 = 13 + 8 = 21

Proof. Use Induction

Base case n=0: 0 = F_0 - 1 = 1 - 1 = 0.
Inductive step: Assume F_n + F_{n-1} = F_{n+1}
Then F_{n+1} + F_n + F_{n-1} - 1 + F_{n+1} = F_{n+2} - 1 (by def. of
Fibonacci numbers)

Theorem. Every group of n people all have
the same name.

Proof. Use Induction

Base case n=1, obvious
Inductive step: Assume the statement is true
for n. If there are n+m people,
number everyone from 1 to n+m.

\[ \{ 1 \} - \text{group 1} \]
\[ \{ 2 \} - \text{group 2} \]

Repeat 1...n have the same
name, and people n+1...n+m have the same name.

The inductive hypothesis: Then any n has
the same name as n and people 1...n.
Everyone has the same name.

Pitfalls
- Proof every group of n people all have the same name.

Proof. Use Induction

Base case n=1, obvious
Inductive step: Assume the statement is true
for n. If there are n+m people,
number everyone from 1 to n+m.

\[ \{ 1 \} - \text{group 1} \]
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Repeat 1...n have the same
name, and people n+1...n+m have the same name.

The inductive hypothesis: Then any n has
the same name as n and people 1...n.
Everyone has the same name.

Theorem. If n ≥ 4, 2^n ≤ 1,2,3...n
(n!)
Strong Induction

- Case: \( P(0) \) - Base case
- Case if \( P(0), P(1), \ldots, P(n) \) are all true, then \( P(n+1) \) is true.

(Theorem setup)

Theorem: Every nonnegative integer at least 2 can be written as a product of primes (only divisible by 1 or itself).

Ex. \( n = 5 \) \( \Rightarrow 15 = 5 \cdot 3 \)
\( n = 7 \) \( \Rightarrow 21 = 3 \cdot 7 \)

Proof: Use strong induction.

- Base case: \( n = 2 \), 2 is prime.
- Induction step: Assume that \( P(k) \) is true for all \( k \leq n \).
  - If \( n \) is prime, \( n \) is done.
  - If \( n \) is not prime, it is divisible by integers \( a \) and \( b \) with \( a \neq n \) and \( b \neq n \).

\( n = a \cdot b \) \( \Rightarrow \frac{a \cdot b}{a} = n \), can be written as a product of primes, therefore so can \( n \).