Lecture 8b: Gases and Fluid flow

Pre-reading:

For this lecture, you need to review the ideal gas law from Chemistry (see below)

Note: The ideal gas law you learned in chemistry is: \( PV = nRT \), where \( n \) = #moles and \( R=8.31\text{J/mol/K} \) is the universal gas constant. Turns out there are many variations of the ideal gas law that mix \( V, n, \) and \( R \) in different ways.

The version we will be using in this lecture is: \( P = \frac{P}{m}k_B T \). We will use this expression to express the density \( \rho \) (mass of a molecule/volume of a molecule) as a function of \( P \).

We can also re-express this as \( PV = Nk_B T \).

Note: \( k_B = 1.38 \times 10^{-23} \text{ kg m}^2/(\text{s}^2 \text{ K}) \), \( N \) is number of molecules, and \( m \) is mass of a gas molecule.

From Collisions to Pressure and the Ideal-Gas Law

We can use the fact that the pressure in a gas is due to the collisions of particles with the walls to make some qualitative predictions. FIGURE 12.8 presents a few such predictions.

Based on the reasoning in Figure 12.8, we expect the following proportionalities:

- Pressure should be proportional to the temperature of the gas: \( p \propto T \).
- Pressure should be inversely proportional to the volume of the container: \( p \propto 1/V \).
- Pressure should be proportional to the number of gas particles: \( p \propto N \).

In fact, careful experiments back up each of these predictions, leading to a single equation that expresses these proportionalities:

\[
p = C \frac{NT}{V}
\]

The proportionality constant \( C \) turns out to be none other than Boltzmann's constant \( k_B \), which allows us to write

\[
pV = Nk_B T
\]

Ideal-gas law, version 1

Equation 12.14 is known as the ideal-gas law.
Learning objectives: After this lecture, you will be able to:

1. Explain why the pressure of a liquid varies linearly with depth.
2. Explain why the pressure in a gas varies exponentially with height.
3. Use the exponential pressure expression to solve problems involving gases.
4. Explain what causes fluids to flow.
5. Explain why the volume flow rate is constant for incompressible fluids and use the expression to solve for the flow characteristic in a pipe of varying dimensions.
Surfactants and Respiratory Distress Syndrome

Surfactant = Surface active agent – a substance that reduces the surface tension of the liquid (compared with the pure liquid).

Common Surfactants: soap, detergent, alcohol

Example: A bead of liquid rests on a solid surface, spreading out on the substrate over an area \( A \). If a surfactant is introduced into the liquid, will the area of contact increase, decrease, or stay the same? Answer: increase.

Try it! You can try the above with a bead of water on a plate. Now add soap (soap is a surfactant)

Crucial medical application: Neonatal respiratory distress syndrome. Lungs in premature babies haven’t started to produce surfactant, so it is difficult to fully inflate the alveoli (air sacs inside lungs). Total surface area is 80 m\(^2\), so without surfactant the surface energy is extremely high. Today, we can treat this condition with synthetic surfactants.

Respiratory distress syndrome was the cause of many infant deaths, including JFK’s son Patrick:
Activity 1: Pressure vs height/depth

Hydrostatic Pressure (Fluids)

\[ P_1 = P_0 + \rho_{\text{water}} gh \]
\[ \rho_{\text{water}} = 1000 \text{ kg/m}^3 \]

Air

Water

\[ P_0 = 1 \text{ atm} = 101.5 \text{ kPa} \]

Barometric Pressure (Air)

\[ P_1 = P_0 e^{\frac{y}{H}} \]
\[ H = \frac{k_B T}{m g} \approx 8 \text{ km} \]
\[ \bar{m} = 29 \text{ amu on Earth} \]

2. Sketch the pressure as a function of \( h \), starting at the surface where the pressure is \( P_0 = 1 \text{ atm} \).

3. Sketch the fluid density (as a function of \( h \) and \( y \), respectively) for each of the two situations above, starting at the surface.

4. One of the plots above increases to the right, while the other decreases, why is that? One plot is straight and the other is curved, what qualitative difference in the fluids might lead to this difference?

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Hint: use the ideal gas law, assuming constant temperature
L8b: Am I getting it? #1

1. The picture shows Mount McKinley in Alaska with an elevation of $h = 6200$ m and Wonder Lake in the foreground with a depth of $85$ m. Which expression corresponds to the pressure difference $\Delta P$ experienced when moving from the bottom of Wonder Lake to the top of Mount McKinley? In other words, what is $\Delta P = P_{Bottom \ of \ Wonder \ Lake} - P_{Top \ of \ McKinley}$? The scale height of the atmosphere is $H = 8$ km, the height of Mount McKinley is $h = 6200$ m, and depth of Wonder lake is $D = 85$ m.

A) $\rho_{water}gD - 1\text{atm} \cdot e^{-h/H}$
B) $1\text{atm} \cdot e^{-h/H} - \rho_{water}gD$
C) $\rho_{water}gD - \rho_{air}gh$
D) $\rho_{air}gh - \rho_{water}gD$
E) $1\text{atm} + \rho_{water}gD - 1\text{atm} \cdot e^{h/H}$
F) $\rho_{water}gD - 1\text{atm} \cdot e^{-h/H} - 1\text{atm}$
Activity 2: Gas flow

So far we have considered only static fluids (i.e. fluids at rest). We will now study fluid dynamics.

1. Fluid or gas can flow due to a "body force" (like gravity), a shear force (like spreading honey), or a pressure difference. Which of these causes air to flow in and out of the lungs?

2. Based on the demonstration with the manometer, what is the approximate pressure in your lungs when you attempt to blow air out of your mouth? (Recall that 1 atm = 101 kPa, and water density is 1000 kg/m$^3$)

Note: once motion has stopped we can use hydrostatics.

Manometer height measurement: _________

3. How, physically, might you create a pressure increase inside your lungs? Hint: idea gas law

The heart: an amazing pressure pump! Pressure drives blood flow through vessels.
Activity 3: Volume flow rate

The volume flow rate \( Q \) (measure in \( \text{m}^3/\text{sec} \)) for incompressible fluids is constant anywhere along a tube, as shown on the right.

\[
Q = \frac{dV}{dt} = A \cdot v
\]

1. Please explain why the volume flow rate HAS TO BE constant, no matter the cross-sectional area \( A \) of a pipe. What would it mean if the flow rate were not constant along the pipe?

2. If the tube is 2 cm in length and the cross-sectional area goes from \( A_1 = 4 \text{cm}^2 \) down to \( A_2 = 1 \text{cm}^2 \), what is the speed \( v_2 \) compared with the incoming speed \( v_1 \)?

3. The volume flow rate of blood leaving the heart to circulate throughout the body is about 5 L/min (8 \( \times \) 10\(^{-5} \) \( \text{m}^3/\text{s} \)) for a person at rest. All this blood eventually must pass through the smallest blood vessels, the capillaries. Microscope measurements show that a typical capillary is 3 \( \mu \text{m} \) in radius, and the blood flows through it at an average speed of 1 mm/sec. Estimate the average number of capillaries in the body.

Bonus! An average capillary is 1 mm long. How far (what distance) would all your capillaries arranged along a straight line run?
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One-Minute Paper

Your name: ____________________________  TF: ____________________________

• Please tell us any questions that came up for you today during lecture. Write “nothing” if no questions(s) came up for you during class.

• What single topic left you most confused after today’s class?

• Any other comments or reflections on today’s class?