Domains of transport
A flow variable that scales well (although it is not always easy to determine) is \( \tau \). This is used in most general transport models in common use.

What might a general transport model look like? The best way to evaluate this question is with a dimensional analysis which will produce a list of dimensionless variables that represent the problem.

This helps because

* the number of dimensionless variables is (usually) 3 less than the full list of variables that we think might be important
* the dimensionless variables often have a useful physical meaning
* Any relation between the dimensionless variables should be general, if we have listed all relevant variables at the start.

Dimensional analysis gives:

\[
q_s = f(\tau, h, D, \rho_s, \rho, \mu, g)
\]

\[
\frac{L^2}{T} = \frac{M}{LT^2}, L, L, \frac{M}{L^3}, \frac{M}{L^3}, \frac{M}{LT}, \frac{L}{T^2}
\]

If \( s = \text{const} \)

\( D << h \)

\& \( S^* \text{ large} \)

\[
q^* = f(\tau^*, S^*, s, D/h)
\]

\[
q^* = \frac{q_s}{\sqrt{(s-1)gD^3}}, \tau^* = \frac{\tau}{(s-1)\rho g D}
\]

\[
S^* = \frac{\sqrt{(s-1)gD^3}}{\mu/\rho} \text{ and } s = \frac{\rho_s}{\rho}
\]

\( q^* \) is dimensionless sediment transport rate

\( \tau^* \) is Shields number

\( S^* \) is dimensionless viscosity

\( s \) is relative density
A typical transport model: 

\[ q^* = a(\tau^* - \tau_c^*)^b \]

Meyer-Peter & Müller: 

\[ q^* = 8(\tau^* - \tau_c^*)^{3/2} \]

One more transport variable, \( W^* \),

One more stress, the reference stress \( \tau_r \),

Convert the M-PM formula to \( W^* \) and incorporate a reference shear stress. First, we divide M-PM by \((\tau^*)^{3/2}\) to get

\[ W^* = \frac{q^*}{(\tau^*)^{3/2}} = \frac{(s-1)gq_s}{(\tau / \rho)^{3/2}} \]

At \( \tau^* = \tau_r^* \), \( W^* = W_r^* = 0.002 \). Dividing by 8, raising both sides to the 2/3 power produces

\[ W^* = 8 \left( 1 - \frac{\tau_c^*}{\tau_r^*} \right)^{3/2} \]

The Difference Between \( \tau_c \) and \( \tau_r \):

\( \tau_c \) : boundary between motion & no motion.
Definable exactly for an individual grain;
definable statistically for a river bed

Hard to measure

\( \tau_r \) : the stress associated with a small,
agreed-upon transport rate \((W^* = 0.002)\)
provides a threshold for a transport relation

Easy to determine from transport observations
The Measurement of $\tau_c$ and $\tau_r$

$\tau_r$ : from transport observations

$\tau_c$ : tracers (painted rocks, magnetic rocks, radio rocks, rock scum) answer the question “did the grain move at all?”
(place tracers, return after flow, measure # moved, repeat for range of flows, develop relation between %entrained, grain size, and flow)

Need large numbers of grains for reliable sample

Need to place “naturally”

The application of $\tau_c$ and $\tau_r$

$\tau_r$ : its purpose is applied: a threshold for a transport relation

It also provides a measurable surrogate for $\tau_c$

$\tau_c$ : sometimes we are interested in the entrainment of individual grains (or the proportion of grains entrained). For example,

“At what discharge do 90% of the surface grains become entrained, thereby providing access to the subsurface and some flushing action?” Or,

“At what discharge will 1% of the surface grains be entrained, thereby indicating that our threshold channel is beginning to fall apart?”
Transport rate is a **steep, nonlinear** function of bed shear stress.

Small error in $\tau$ can produce big error in $q_s$.

Three things make it difficult to accurately estimate $\tau$:

1. Accelerations in *unsteady, nonuniform* (i.e. real) flow
2. Only a portion of the total $\tau$ acts to transport sediment
3. $\tau$ varies locally. Because transport is a nonlinear fn of $\tau$, estimates based on total $\tau$ will be in error.
2006 PACIFIC CREEK SEDIMENT TRANSPORT RESULTS:

- Significant transport begins ~30 m³/s
- Measured transport rates range from 1 – 1300 g/m/s
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Example calculation using Meyer-Peter and Muller for a channel with slope $S$ =0.002, roughness $n$ = 0.025, and width $b$ = 15m. The solid curve uses $\tau^* = 0.045$ and $D = 45$ mm. The dashed curve uses $\tau^* = 0.045$ and $D = 30$ mm.

Note that at discharge $Q = 55$ cms, one curve indicates zero transport and the other a transport rate of 80,000 kg/hr.
Factors that induce degradation below dams:
- Reduced sediment supply
- Fining of sediment supply

Factors that induce aggradation below dams:
- Reduced floods or total stream flow
- Coarsening of sediment supply

Quantitative analysis of perturbations

Borland’s illustration of Lane’s (1955) concept, drawn by Vitaliano
The Lane Balance, quantified almost 40 yrs ago by Henderson (1966, Open Channel Flow)

Einstein-Brown depth-slope continuity Chezy

\[ \frac{q_s^*}{q_s} \sim \tau^{3/2} \]
\[ \tau \sim R \]
\[ q = U R \]
\[ U \sim (R S)^{0.5} \]

\[ q_s^* \sim \tau^{3/2} / D^{1.5} \]
\[ q_s \sim (R S)^{3/2} / D^{1.5} \]

\[ q_s \sim (q^2 S^2) / D^{1.5} \]

\[ S \sim (q_s^{0.5} D^{0.75}) / q \]

\[ \frac{S_{\text{post}}}{S_{\text{pre}}} \propto \sqrt{q_s_{\text{post}} / q_s_{\text{pre}}} \left( \frac{q_{\text{pre}}}{q_{\text{post}}} \right) \left( \frac{D_{\text{post}}}{D_{\text{pre}}} \right)^{0.75} \]

\[ q_s^* \sim q_s / D^{1.5} \]
\[ \tau^* \sim \tau / D \]
\[ R \sim h \]

Interpretation, for evaluating stream history

Steady state: sediment supply balanced by transport capacity. Slope is stable.

Increase sediment supply
Sediment supply > transport capacity
sediment accumulates

Increase water supply
Sediment supply < transport capacity
sediment evacuates

\[ \frac{S_2}{S_1} = \sqrt{q_{b2} / q_{b1}} \left( \frac{D_2}{D_1} \right)^{3/4} \left( \frac{q_1}{q_2} \right) \]
Figure 9. Likely channel response (scour or aggradation) to changes in water and sediment supply from $q_0$ to $q_f$. The solid line represents the solution of equation 6 for no change in the grain size of the sediment supply ($D_0/D_i = 1$). Solutions are also shown for a factor of two change in grain size, $D_0/D_i = 1$. Channels falling below or to the right of the line would tend to steepen through preferential deposition; the slope of channels above and to the left of the line would decrease through preferential scour. Approximate water and sediment supply for different periods in northeast Puerto Rico (Table 2) suggest a sequence of aggradation followed by degradation.