Theorem. In a grid (m x n) where each cell contains a black or blue colored point, prove that if there are at least 3 black points per row and column, then there are at least 2 black points in the same row or column.

Proof. Let points $(i, j)$, $(i, k)$, and $(k, j)$ be black for some $i, j, k$. Then by the pigeonhole principle, there exists a cell $c$ in row $i$ that contains at least one black point. Similarly, there exists a cell $d$ in column $j$ that contains at least one black point. Thus, there are at least 2 black points in the same row or column.

Theorem. 5 points in a 1x1 square, then each 2 points of distance less than $\frac{1}{2}$.

Proof. Divide 1x1 square into $\frac{1}{2}$ areas $\frac{1}{2}$

At most 4 points in 4 side. By pigeonhole principle, there exists a point in $\frac{1}{2}$

Then distance between 2 points is less than $\frac{1}{2}$

(by symmetry)

Theorem. $\lambda + \mu = \lambda' + \mu'$

Then both exist, $\lambda = \mu = \lambda'$

$\lambda, \mu, \lambda', \mu'$

Proof. $\lambda$ and $\lambda'$ are both squares, so $\lambda = \mu = \lambda'$

$\lambda + \lambda' = \lambda + \mu' = \mu'$

Now $\lambda + \lambda' = \mu'$

$\lambda + \lambda' = \lambda + \mu' = \lambda'$

$\lambda + \lambda' = \lambda + \mu'$

$\lambda + \lambda' = \lambda$