Chapter 6 Assessment

1. Choose one of the following exercises:

   (a) Find the work done in moving a particle from the point $(3, 0, 0)$ to the point $(0, \pi/2, 3)$ in the force field $\mathbf{F}(x, y, z) = \langle z, x, y \rangle$. (a) along a straight line; (b) along the helix $\mathbf{r}(t) = \langle 3\cos(t), t, 3\sin(t) \rangle$.

   (b) Use Green's Theorem to evaluate

   $$\oint_C \sqrt{1+x^3}\, dx + 2xy\, dy$$

   where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$ oriented counterclockwise.

   (c) Let $\mathbf{F}(x, y, z) = \langle 4x^3y^2 - 2xy^3, 2x^4y - 3x^2y^2 + 4y^3 \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve $\mathbf{r}(t) = \langle 1 + \sin(\pi t), 2t + \cos(\pi t) \rangle$ with $0 \leq t \leq 1$.

2. Choose one of the following exercises:

   (a) Show that there is no vector field, $\mathbf{G}$, such that $\nabla \times \mathbf{G} = \langle 2x, 3yz, -xz^2 \rangle$.

   (b) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle xz, -2y, 3x \rangle$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.

   (c) Use Stokes' Theorem to evaluate $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle x^2yz, yz^2, z^3e^{xy} \rangle$, S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$, and S is oriented upward.

3. Choose one of the following exercises:

   (a) Compute the outward flux of

   $$\mathbf{F}(x, y, z) = \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}$$

   through the ellipsoid $4x^2 + 9y^2 + 6z^2 = 36$.

   (b) Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x^3, y^3, z^3 \rangle$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.

   (c) Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle z^2x, y^3/3 + \tan(z), x^2z + y^2 \rangle$ and S is the top half of the sphere $x^2 + y^2 + z^2 = 1$. [*Hint:* Note that S is not a closed surface. First, compute the integrals over S_1 and S_2, where S_1 is the disk $x^2 + y^2 \leq 1$, oriented downward, and $S_2 = S \cup S_1$ (closed, div thm).]