Exponential Functions

Growth

Decay
Exponential function – A function of the form $y=ab^x$, where $b>0$ and $b\neq 1$.

Step 1 – Make a table of values for the function.

$$y = 3^x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3^x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$3^{-2} = \frac{1}{3^2}$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>$0$</td>
<td>$3^0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$3^2$</td>
<td>$9$</td>
</tr>
<tr>
<td>$3$</td>
<td>$3^3$</td>
<td>$27$</td>
</tr>
</tbody>
</table>
Now that you have a data table of ordered pairs for the function, you can plot the points on a graph.

\[ (-2, \frac{1}{9}) \quad (0,1) \quad (2,9) \]

Draw in the curve that fits the plotted points.
Domain – The collection of all input values of a function. These are usually the “x” values.

Range – The collection of all output values of a function. These are usually the “y” values.

Describe the domain and range of the function $y = -5^x$.

**Domain** – The domain of the function is all real numbers since the function is defined for all $x$-values.

**Range** – The range of the function is all negative real numbers.
If a quantity increases by the same proportion $r$ in each unit of time, then the quantity displays exponential growth and can be modeled by the equation

$$y = C (1 + r)^t$$

Where

$C = \text{initial amount}$

$r = \text{growth rate (percent written as a decimal)}$

$t = \text{time}$

$(1+r) = \text{growth factor \ where } 1 + r > 1$
You deposit $1500 in an account that pays 2.3% interest compounded yearly,

1) What was the initial principal (P) invested?

2) What is the growth rate (r)? The growth factor?

3) Using the equation $A = P(1+r)^t$, how much money would you have after 2 years if you didn’t deposit any more money?

1) The initial principal (P) is $1500.

2) The growth rate (r) is 0.023. The growth factor is 1.023.

3) $A = P(1 + r)^t$

$A = 1500(1 + 0.023)^2$

$A = $1569.79
If a quantity decreases by the same proportion \( r \) in each unit of time, then the quantity displays exponential decay and can be modeled by the equation

\[
y = C (1 - r)^t
\]

Where

- \( C \) = initial amount
- \( r \) = growth rate (percent written as a decimal)
- \( t \) = time
- \((1 - r)\) = decay factor where \( 1 - r < 1 \)
You buy a new car for $22,500. The car depreciates at the rate of 7% per year,

1) What was the initial amount invested?
2) What is the decay rate? The decay factor?
3) What will the car be worth after the first year? The second year?

1) The initial investment was $22,500.
2) The decay rate is 0.07. The decay factor is 0.93.
3) \[ y = C (1 - r)^t \]

\[ y = 22,500(1 - 0.07)^1 \]
\[ y = $20,925 \]
\[ y = 22,500(1 - 0.07)^2 \]
\[ y = $19,460.25 \]
1) Make a table of values for the function \( y = \left( \frac{1}{6} \right)^x \) using \( x \)-values of \(-2, -1, 0, 1, \) and \(2\). Graph the function. Identify the domain and range of the function. Does this function represent exponential growth or exponential decay?

2) Your business had a profit of $25,000 in 1998. If the profit increased by 12% each year, what would your expected profit be in the year 2010? Identify \( C, t, r, \) and the growth factor. Write down the equation you would use and solve.

3) Iodine-131 is a radioactive isotope used in medicine. Its half-life or decay rate of 50% is 8 days. If a patient is given 25mg of iodine-131, how much would be left after 32 days or 4 half-lives? Identify \( C, t, r, \) and the decay factor. Write down the equation you would use and solve.
The **domain** of this function is the set of all real numbers.

The **range** of this function is the set of all positive real numbers.

This function represents exponential decay.
Problem 2

\[ C = \$25,000 \]
\[ T = 12 \]
\[ R = 0.12 \]

*Growth factor* = 1.12

\[ y = C (1 + r)^t \]
\[ y = \$25,000(1 + 0.12)^{12} \]
\[ y = \$25,000(1.12)^{12} \]
\[ y = \$97,399.40 \]
Problem 3

\[ C = 25 \text{ mg} \]
\[ T = 4 \]
\[ R = 0.5 \]

Decay factor = 0.5

\[ y = C (1 - r)^t \]
\[ y = 25mg(1 - 0.5)^4 \]
\[ y = 25mg(0.5)^4 \]
\[ y = 1.56mg \]