INSTRUCTIONS
You may begin the exam when ready.

This is for practice. Do your best.

Unless otherwise stated, you must justify your solutions to receive full credit. Work that is illegible may not be graded. Work that is scratched out will not be graded.

It is fine to leave your answer in a form such as $\ln(0.02)$ or $\sqrt{123412}$ or $(1341)^4(1231)^{-1}$. However, if an expression can be easily simplified (such as $e^{\ln(0.02)}$ or $\cos \pi$), you should simplify it.

The Honor Principle requires that you neither give nor receive any aid on this exam. This exam is open book (you may use only the textbook, notes, and videos from canvas). You may not use calculators, software, or the internet outside of canvas. You may ask the instructors for clarification on problems through e-mail. While the exam is for practice, please work in conditions as if it was the actual exam.

The exam has been created with the intended length of 2 hours.

Good luck!
Multiple Choice Questions

For problems 1-7, no justification is required.

(1) The derivative of function $f(x)$ can be expressed as follows:

$$f'(x) = \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}$$

(a) True  
(b) False

(2) The following function $f(x)$ has jump discontinuity at $x = 0$

$$f(x) = \begin{cases} 
-x + 1 & x > 0 \\
2 & x = 0 \\
1 + 2x & x < 0 
\end{cases}$$

(a) True  
(b) False

(3) What is the average rate of change of $f(x) = \ln x^2$ between $x = 1$ and $x = 2$?

(a) $\ln 4$  
(b) 0  
(c) $\ln 2$  
(d) 1  
(e) 2

(4) If a function is continuous at $x = a$, then it is also differentiable at $x = a$.

(a) True  
(b) False
(5) Consider the two graphs of functions

(a) Is \( f(x) \) differentiable at \( x = -2 \)?

(b) Find \( (g \circ f)'(0) \).

(c) Find \( (f \circ g)'(1) \).

(d) Find \( (f \cdot g)'(1) \).
(6) Match the following expressions to their limits by drawing a line. Note, more than one expression can have the same limit.

\[
\begin{align*}
\lim_{{x \to -8}} f(x) &= -6 \\
\lim_{{x \to -2}} f(x) &= -3 \\
\lim_{{x \to 6}} f(x) &= 0 \\
\lim_{{x \to 10^+}} f(x) &= 3 \\
\end{align*}
\]

No Limit

(7) Match the following points to their type of discontinuity by drawing a line.

\[
\begin{align*}
x = -8 & \quad \text{Infinite Discontinuity} \\
x = -2 & \quad \text{Removable Discontinuity} \\
x = 6 & \quad \text{Continuous} \\
x = 10 & \quad \text{Jump Discontinuity} \\
x = -4 &
\end{align*}
\]
Long Answer Questions

For problems 8-11, please show all your work to receive full credit.

(8) (a) Let \( \{a_n\} = \{\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots \} \)

Assuming \( \{a_n\} \) continues in the same pattern, which of the following is a general form for \( a_n \)? No justification needed.

(i) \( \frac{1}{n} \)

(ii) \( \frac{n}{2n - 1} \)

(iii) \( \frac{n + 1}{n + 3} \)

(iv) \( \frac{n}{n + 2} \)

(b) What does \( \{a_n\} \) converge to?

(c) Let \( \epsilon = \frac{1}{4} \). Find an \( N \) that satisfies the definition of convergence for this \( \epsilon \). (Note that the definition of a sequence converging to \( L \) is that for any \( \epsilon > 0 \) there is an \( N \) such that \( |a_n - L| < \epsilon \) for any \( n \geq N \).) Be sure to justify your answer.
(9) Prove that there is at least a number $x$ between 0 and $\pi$ such that $\tan(x) = x + \frac{1}{3}$. 
(10) (a) Find the value of $k$ for which $f(x)$ is continuous.

$$f(x) = \begin{cases} 
  x^2 & x < k \\
  2x - 1 & x \geq k
\end{cases}$$

(b) Is $f(x)$ differentiable at $x = k$ (the one you found in part (a))? Why or why not?
(11) A student forgot to justify the computation of the following limit

\[ \lim_{x \to 3} \frac{\sin(x - 3 + \frac{\pi}{2})}{\cos(-\pi x)}. \]

Below is the student’s computation. Explain what theorem the student used at each step, and explain if the student respected the conditions for using these theorems.

\[
\begin{align*}
\lim_{x \to 3} \frac{\sin(x - 3 + \frac{\pi}{2})}{\cos(-\pi x)} &= \frac{\lim_{x \to 3} \sin(x - 3 + \frac{\pi}{2})}{\lim_{x \to 3} \cos(-\pi x)} \\
&= \frac{\sin(\lim_{x \to 3} x - 3 + \frac{\pi}{2})}{\cos(\lim_{x \to 3} -\pi x)} \\
&= -1
\end{align*}
\]
(This page is intentionally left blank in case you need extra space for any of the problems. If you use this page for a particular problem, it is essential that you make a note on the page where the problem appears, indicating that your work is continued here.)