SMART START

A square has an area of 144 in\(^2\). What is the length, to the nearest inch, of the diagonal?

- 15 inches
- 16 inches
- 17 inches
- 18 inches

\[ a^2 + b^2 = c^2 \]
\[ 12^2 + 12^2 = c^2 \]
\[ 144 + 144 = c^2 \]
\[ 288 = c^2 \]
\[ 17 \]

Brianna starts from home and walks 6 km west. She then takes a left and walks 8 km south.

Using the shortest distance between her home and current location, how far is Brianna from her home in the scenario described?

- \( \sqrt{2} \) km
- 21 km
- 10 km
- 14 km

\[ a^2 + b^2 = c^2 \]
\[ 6^2 + 8^2 = c^2 \]
\[ 36 + 64 = c^2 \]
\[ 100 = c^2 \]
\[ c = 10 \]
Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. \(27y^2 + 18y\)

Find the GCF of each term.
\[
27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y
\]
\[
18y = 2 \cdot 3 \cdot 3 \cdot y
\]
GCF = 3 \cdot 3 \cdot y or 9y

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to factor out the GCF.
\[
27y^2 + 18y = 9y(3y) + 9y(2)
\]
\[
= 9y(3y + 2)
\]

b. \(-4a^2b - 8ab^2 + 2ab\)

Factor each term.
\[
-4a^2b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b
\]
\[
-8ab^2 = -1 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b
\]
GCF = 2 \cdot a \cdot b or 2ab

Rewrite each term using the GCF.
\[
-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1)
\]
\[
= 2ab(-2a - 4b + 1)
\]

Guided Practice

1A. \(15w - 3v\) \(3(5w - v)\)

1B. \(7u^2t^2 + 21ut^2 - ut\) \(ut(7ut + 21t - 1)\)
Example 2  Factor by Grouping

Factor $4qr + 8r + 3q + 6$.

Original expression

$4qr + 8r + 3q + 6$

Group terms with common factors.

$= (4qr + 8r) + (3q + 6)$

Factor the GCF from each group.

$= 4(r + 2) + 3(q + 2)$

Notice that $(q + 2)$ is common in both groups, so it becomes the GCF.

$= (4r + 3)(q + 2)$

Distributive Property

Guided Practice

Factor each polynomial.

2A. $mn + 5n - r - 5 \quad (r + 5)(n - 1)$

2B. $3np + 15p - 4n - 20 \quad (n + 5)(3p - 4)$

\[
(4qr + 8r) + (3q + 6) \\
4r(q + 2) + 3(q + 2) \\
(q + 2)(4r + 3)
\]
Example 3  Factor by Grouping with Additive Inverses

Factor $2mk - 12m + 42 - 7k$.

$2mk - 12m + 42 - 7k$

$= (2mk - 12m) + (42 - 7k)$

$= 2m(k - 6) + 7(6 - k)$

$= 2m(k - 6) + 7[(-1)(k - 6)]$

$= 2m(k - 6) - 7(k - 6)$

$= (2m - 7)(k - 6)$

Guided Practice

Factor each polynomial.

3A. $c - 2cd + 8d - 4$

3B. $3p - 2p^2 - 18p + 27$

$(c - 2cd) + (8d - 4)$

C$(1 - 2d) + 4(2d - 1)$

C$(1 - 2d) + 4(-2d + 1)$

C$(1 - 2d) + 4(1 - 2d)$

$(c - 2cd)(1 - 2d)$

$(2m - 7)(k - 6)$
Example 4 Solve Equations

Solve each equation. Check your solutions.

a. \((2d + 6)(3d - 15) = 0\)
   
   \[
   \begin{align*}
   2d + 6 &= 0 & \text{or} & & 3d - 15 &= 0 \\
   2d &= -6 & & 3d &= 15
   \end{align*}
   \]
   
   Zero Product Property
   
   \[
   \begin{align*}
   d &= -3 & & d &= 5
   \end{align*}
   \]
   
   Divide.
   
   The roots are \(-3\) and \(5\).

   **CHECK** Substitute \(-3\) and \(5\) for \(d\) in the original equation.

   \[
   \begin{align*}
   (2d + 6)(3d - 15) &= 0 \\
   [2(-3) + 6][3(-3) - 15] &= 0 \\
   (-6 + 6)(-9 - 15) &= 0 \\
   (0)(-24) &= 0
   \end{align*}
   \]
   
   Solve each equation.

   \[
   0 = 0 \checkmark
   \]

   b. \(c^2 = 3c\)
   
   \[
   \begin{align*}
   c^2 &= 3c & \text{Original equation} \\
   c^2 - 3c &= 0 & \text{Subtract 3c from each side to get 0 on one side of the equation.} \\
   c(c - 3) &= 0 & \text{Factor by using the GCF to get the form } ab = 0. \\
   c &= 0 & \text{or} & & c - 3 &= 0 \\
   & & \text{Zero Product Property} \\
   c &= 3 & \text{Solve each equation.} \\
   \end{align*}
   \]
   
   The roots are \(0\) and \(3\).

   Guided Practice

   4A. \(3n(n + 2) = 0\) 
   4B. \(8b^2 - 40b = 0\) 
   4C. \(x^2 = -10x\)
\( 3n(n+2) = 0 \)

\( 3n = 0 \quad n+2 = 0 \)

\( n = 0 \quad n = -2 \)

\( 8b^2 - 40b = 0 \)

\( 8b(b - 5) = 0 \)

\( b = 0 \quad b = 5 \)

\( x^2 = -10x \)

\( x^2 + 10x = 0 \)

\( x(x + 10) = 0 \)

\( x = 0 \quad x = -10 \)
### Key Concept: Factoring $x^2 + bx + c$

**Words**
To factor trinomials in the form $x^2 + bx + c$, find two integers, $m$ and $p$, with a sum of $b$ and a product of $c$. Then write $x^2 + bx + c$ as $(x + m)(x + p)$.

**Symbols**
$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$.

**Example**
$x^2 + 6x + 8 = (x + 2)(x + 4)$, because $2 + 4 = 6$ and $2 \cdot 4 = 8$. 
Example 1  \( b \) and \( c \) are Positive

Factor \( x^2 + 9x + 20 \).

In this trinomial, \( b = 9 \) and \( c = 20 \). Since \( c \) is positive and \( b \) is positive, you need to find two positive factors with a sum of 9 and a product of 20. Make an organized list of the factors of 20, and look for the pair of factors with a sum of 9.

<table>
<thead>
<tr>
<th>Factors of 20</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 20</td>
<td>21</td>
</tr>
<tr>
<td>2, 10</td>
<td>12</td>
</tr>
<tr>
<td>4, 5</td>
<td>9</td>
</tr>
</tbody>
</table>

\[
x^2 + 9x + 20 = (x + m)(x + p)
= (x + 4)(x + 5)
\]

**CHECK** You can check this result by multiplying the two factors. The product should be equal to the original expression.

\[
(x + 4)(x + 5) = x^2 + 5x + 4x + 20
= x^2 + 9x + 20 \checkmark
\]

**Guided Practice**

Factor each polynomial.

1A. \( d^2 + 11d + 24 \)  

1B. \( 9 + 10t + t^2 \)

\[ (x + 4)(x + 5) \]

\[ x^2 + 9x + 20 \]
(A) $d^2 + 11x + 24$

$\frac{24}{3} = 8$

(B) $9 + 10t + t^2$

$\frac{t^2 + 10t + 9}{3}$

$(t+1)(t+9)$
Example 2  \( b \) is Negative and \( c \) is Positive

Factor \( x^2 - 8x + 12 \). Confirm your answer using a graphing calculator.

In this trinomial, \( b = -8 \) and \( c = 12 \). Since \( c \) is positive and \( b \) is negative, you need to find two negative factors with a sum of \(-8\) and a product of \(12\).

<table>
<thead>
<tr>
<th>Factors of 12</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, -12)</td>
<td>(-13)</td>
</tr>
<tr>
<td>(-2, -6)</td>
<td>(-8)</td>
</tr>
<tr>
<td>(-3, -4)</td>
<td>(-7)</td>
</tr>
</tbody>
</table>

The correct factors are \(-2\) and \(-6\).

\[
x^2 - 8x + 12 = (x + m)(x + p) \quad \text{Write the pattern.}
= (x - 2)(x - 6) \quad m = -2 \text{ and } p = -6
\]

CHECK Graph \( y = x^2 - 8x + 12 \) and \( y = (x - 2)(x - 6) \) on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly.

Guided Practice

Factor each polynomial.

2A. \( 21 - 22m + m^2 \)

2B. \( w^2 - 11w + 28 \)
Example 3  \(c\) is Negative

Factor each polynomial. Confirm your answers using a graphing calculator.

a. \(x^2 + 2x - 15\)

In this trinomial, \(b = 2\) and \(c = -15\). Since \(c\) is negative, the factors \(m\) and \(p\) have opposite signs. So either \(m\) or \(p\) is negative, but not both. Since \(b\) is positive, the factor with the greater absolute value is also positive.

List the factors of \(-15\), where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

<table>
<thead>
<tr>
<th>Factors of (-15)</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, 15)</td>
<td>14</td>
</tr>
<tr>
<td>(-3, 5)</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
x^2 + 2x - 15 = (x + m)(x + p)
= (x - 3)(x + 5)
\]

**CHECK** \((x - 3)(x + 5) = x^2 + 5x - 3x - 15\)
\[
= x^2 + 2x - 15 \checkmark
\]

The correct factors are \(-3\) and \(5\).

Write the pattern.
\(m = -3\) and \(p = 5\)

FOIL Method
Simplify.

\((-1, 15\)
\(-3\)
\((-3)(x+5)\)
b. \( x^2 - 7x - 18 \)

In this trinomial, \( b = -7 \) and \( c = -18 \). Either \( m \) or \( p \) is negative, but not both. Since \( b \) is negative, the factor with the greater absolute value is also negative.

List the factors of \(-18\), where one factor of each pair is negative. Look for the pair of factors with a sum of \(-7\).

<table>
<thead>
<tr>
<th>Factors of (-18)</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, (-18)</td>
<td>(-17)</td>
</tr>
<tr>
<td>2, (-9)</td>
<td>(-7)</td>
</tr>
<tr>
<td>3, (-6)</td>
<td>(-3)</td>
</tr>
</tbody>
</table>

The correct factors are 2 and \(-9\).

\[
x^2 - 7x - 18 = (x + m)(x + p)
= (x + 2)(x - 9)
\]

Write the pattern. \( m = 2 \) and \( p = -9 \).

**CHECK** Graph \( y = x^2 - 7x - 18 \) and \( y = (x + 2)(x - 9) \) on the same screen.
3A. \( y^2 + 13y - 48 \)

\[
\begin{array}{c}
\frac{+48}{2+8}\\
\frac{3+16}{4+12}\\
0+8
\end{array}
\]

\((y-3)(y+16)\)

3B. \( r^2 - 2r - 24 \)

\[
\begin{array}{c}
\frac{+24}{2+12}\\
\frac{3+8}{4-6}
\end{array}
\]

\((r+4)(r-6)\)
### Example 4 Solve an Equation by Factoring

Solve \( x^2 + 6x = 27 \). Check your solutions.

| \( x^2 + 6x = 27 \) | Original equation |
| \( x^2 + 6x - 27 = 0 \) | Subtract 27 from each side. |
| \( (x - 3)(x + 9) = 0 \) | Factor. |
| \( x - 3 = 0 \) or \( x + 9 = 0 \) | Zero Product Property |
| \( x = 3 \) \( \quad \) \( x = -9 \) | Solve each equation. |

The roots are 3 and -9.

**CHECK** Substitute 3 and -9 for \( x \) in the original equation.

- \( x = 3 \):
  
  \[ (3)^2 + 6(3) = 9 + 18 = 27 \]

- \( x = -9 \):
  
  \[ (-9)^2 + 6(-9) = 81 - 54 = 27 \]

\[ 27 = 27 \checkmark \]

### Guided Practice

Solve each equation. Check your solutions.

4A. \( z^2 - 3z = 70 \)

4B. \( x^2 + 3x - 18 = 0 \)