Advice for Homework and Exams

Solving Homework Problems

It is expected that you will spend some time being stuck when working on homework problems. Being stuck can be uncomfortable, but it is necessary to learn - if you are never stuck it is probably because you are already familiar with the material. If you are stuck, you can try the following strategies to be unstuck.

Identify the relevant material

Usually homework problems are assigned with some specific material covered in class or in the text in mind. Sometimes it's not clear what this is. If so, try to think through what was covered in recent lectures and see if there's any way to connect it with the problem you're working on.

Once you've identified the relevant material, make sure you actually know it! Look over your notes and the corresponding section in the text, and pay particular attention to the proofs of theorems. Oftentimes the techniques used for the proofs we do in class or in the text can also be used to solve problems in the homework. Other times, you might realize that the theorems we proved can be used to simplify the problem or transform it into something more manageable.

Try out examples

Examples are not proofs. But they can often give you hints as to what exactly is going on, and give you ideas for how to generalize. For example, if a problem asks you to prove a statement for all non-negative integers n, you can start by proving the statement for $n = 1$, $n = 2$, $n = 3$, and so on. If a problem asks you to prove a statement for all random variables, you can try specific types of random variables. After a while you might find a pattern.

Be careful to not be too hasty in generalizing, as it can sometimes be easy to pick examples that aren't representative and come up with a solution that is incorrect. When picking examples, try to look for a wide variety of examples for which the same proof techniques won't necessarily work. After writing your proof, try to see if you can come up with an example that breaks your proof, that is, an example that shows that the logic you used in your proof is incorrect.

Solve a simpler problem

See if adding more assumptions or restrictions makes the problem more approachable. For example, if a problem asks you to prove a statement for all non-negative integers, try proving the problem for all even integers, or all prime numbers if you think that might be easier to solve.

Once you have done this, go back to the original problem, and try to see what the missing link is. If you are able to solve the problem for even integers and not odd integers, for example, try to see what it is about odd numbers that makes the problem harder, or what parts of your proof don't work for odd numbers. This can give you something more focused to work on.

Solve a different problem

Most problems will have a set of assumptions, and a conclusion that you need to show follows from the set of assumptions. If you're not able to reach the conlusion asked of you, try to see what conlusions you can prove. It might turn out that the conclusions you can prove are just intermediate steps in the final proof.

Alternatively, you can try to see what assumptions you can come up with that will lead you to the necessary conclusion. This might also be an intermediate step in the final proof. But be careful to avoid starting by assuming that the conclusion holds. This can be helpful in thinking about the meaning behind your proof, but it is usually not helpful in actually coming up with the proof itself.

Discuss with your group

See what other people in your group think and have tried. It could be that one person has solved one version of a simpler problem, and another person has solved another version of a simpler problem, and together that you've solved the entire problem. Or, it could be that one of your group members points out a technique used in a proof in the text or lecture that inspires you to try it out yourself.

Rinse and repeate

Sometimes you might have to go through many iterations of the above before you actually solve the problem. After solving a simpler problem, you might realize that you didn't understand the material as well as you thought and spend time reading over your notes and the text again. Or after solving a different problem, you might realize that there are a lot of examples that you haven't yet tried, and you might spend some time examining new examples.

Studying for exams

When studying for exams, the usual advice still applies. However, the following strategies also work well for proof-based math courses.

Understand the motivation behind proofs

In addition to understanding the proof itself, try to figure out how the proof came about. If you were given the theorem as a homework problem, how would you approach it? What examples might you try, and where might you get stuck? We will rarely ask you to reprove theorems you've already seen, but thinking through these steps can help you come up with additional strategies on how to approach problems in exams.

Understand the assumptions in theorems

Mathematicians are usually careful with their assumptions for theorems. If the theorem could apply to other cases, they would be included. For example, if a theorem only applies for even integers or prime numbers, it is likely that the proof doesn't work for odd integers or composite numbers. This means two things. First, the proof should use that assumption somewhere in the theorem, that is, one of the statements in the proof should be false if we remove the assumption. Second, it is likely that there are counterexamples if the assumptions are relaxed. Try to find both of these.

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Consider modified versions of theorems

Try to see what happens when you modify the statements of theorems you've seen in class or in the text. Does the same proof still work? Why or why not? This is one way to get additional practice problems. For example, we saw in class a proof that $\sqrt{2}$ is irrational. What happens if we want to prove that $\sqrt{3}$, $\sqrt{4}$, or $\sqrt{6}$ are irrational?