Boolean Functions

1. Summary
   - Spectral Concentration
   - Decision Trees

2. Q+ A

3. Problems

Given query/random access to \( f: \{-1, 1\}^n \rightarrow \{-1, 1\} \), output \( h: \{-1, 1\}^n \rightarrow \{-1, 1\} \), does \( h(f, \tilde{f}) \leq \varepsilon \).

Assume \( f \in \mathcal{C} \), \( \mathcal{C} \) some class of functions.

- Ex: \( \mathcal{C} = \) linear functions, \( n \) queries.

Decision trees - size vs. \( \varepsilon \) losses

- \( \mathcal{C} = \) size of decision tree

- \( \mathcal{C} = \) depth of longest root-to-leaf path

Then, if \( f \) is \( \varepsilon \)-concentrated on \( \mathcal{F} \), can learn \( f \) to error \( \varepsilon \), in time \( \text{poly}(n, \varepsilon^{-1/2}) \) queries/linear.

- \( \mathcal{F} \) - collection of subclasses of \( \mathcal{F} \)

- \( f \) is \( \varepsilon \)-concentrated on \( \mathcal{F} \) if \( \sum f(x) \leq \varepsilon \) for all \( f \in \mathcal{F} \).

Suppose \( f \) is \( \varepsilon \)-concentrated up to degree \( k \).

- \( \varepsilon = 3 N S_k(f) \)

Using ideas from Markov's inequality

Pens: \( f \) all weighted majority, \( N S_k(f) \leq 1/k \).

Decision trees - decompose into sum of polynomials

- depth \( k \)

- size \( \sum \binom{n}{s} \leq 2^k \)

- degree \( k \)

- \( \delta \) (dt non-zero Fourier)

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Example:

- \( f(S) = \delta_{S1}, \delta \leq 2^k \), \( f(S) \leq \text{integer} \)