En 2. Permutations/Combinations

**Announcements**
- Plan for midterm/final

**Generalized Product Rule**
- If \( S \) is a set of length-\( k \) sequences (of form \( (s_1, s_2, ..., s_k) \)),
  - \( n_1 \) is \# of choices for \( s_1 \)
  - \( n_2 \) is \# of choices for \( s_2 \), given \( s_1 \)
  - \( n_k \) is \# of choices for \( s_k \), given \( s_1, s_2, ..., s_{k-1} \).
- \[ |S| = n_1 \cdot n_2 \cdot ... \cdot n_k \]

**Division Rule**

**Definition**
- \( f: A \rightarrow B \), is \( k \) to \( 1 \) if \( |f^{-1}(b)| = k \) for every \( b \in B \).
- \( |A|^k \geq |B| \)

**Division Rule**
- If \( f: A \rightarrow B \) is \( k \) to \( 1 \),
  - then \( |A|^k \geq |B| \)

Then \# of sequences of length \( k \) from a set \( S \) of size \( k \), no two same, \( |S|^k \), is \( \frac{|S|^k}{k!} \).

**Proof**
- A set of permutations of \( \pi \) is a permutation of \( \pi \) of length \( k \).
- \[ |f^{-1}(b)| = \# \text{ of permutations of } S \setminus b \]

- By division rule:
  - \( |S| \cdot |b| = k! \)
  - \( |\pi| = (\pi) \cdot |\pi| = k! \)

**Visions as a set**
- \( \{ b \mid b \in S \} \) for some \( S \)

**Sorting**
- Permutation \( \rightarrow \) sorted list
- A sequence of swaps

**Sorting algorithm must do a sort sequence of**
- \# of swaps for each permutation
  - 1 steps for each, \( \sum_{i=1}^{k} \frac{1}{k} \)
- \# of permutations is \( \pi(k) \)
- \( |S| \cdot |b| = k! \)

**Example**
- \( |S|^k \geq |B| \)
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Thm. # of subsets of size $k$ of $S$ s.t. $|S|=k$

is $\binom{k}{k} = \binom{k}{k}$ ($\#$ of ways to choose $k$ elements from a set of size $k$)

Proof. $A$: set of length $k$ sequences from $S$, no two are same.

$\mathcal{B}$: subsets of $S$ of size $k$

$f: A \rightarrow \mathcal{B}$, $f(a)$ a word in $a$.

$f^*(b)$ = set of permutations of $b$

$|f^*(b)| = k!$, by division rule, $|\mathcal{B}| = \frac{k!}{k!}$

$|A| (k-1)!$

$\Box$

Thm. # of sequences in $\{0,1\}^n$ of length $2n$ is

# of subsets of $\{0,1\}^n$ of size $2\times n = \binom{2n}{2}$

Proof. $B$: $\{0,1\}^n$ of length $2n$.

$C$: set of words of $\{0,1\}^n$ of size $2n$.

$f: B \rightarrow C$, $f(a) = \{a\}$.

if $a \in C$, $f^*(c) = \{c_1, b, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$

$c = \{c_1, c_2, c_3\}$, $b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8$ not all $0$.

Therefore, $|f^*(c)| = 2$, and $f$ is bijection, $|\mathcal{B}| = 10^n$.

$\Box$