Updated Course Goals:
Component Skills & Holistic Goals.

Amath 352
Applied Linear Algebra

Winter 2020

1 Course Component Skills

These are very specific, targeted skills to measure students' understanding of the material. Each of the 5 units of study will have its own set of component skills, and the material will be presented in order of these skills. I will indicate which component skills will be tested on upcoming quizzes, and homeworks will clearly label which questions relate to which component skills. There should (hopefully) be no surprises! For more, see **Grading** in the syllabus.

Unit 1. Matrix Arithmetic & Introduction to MATLAB

S1.1 Solve a given linear system by putting it into triangular form and using backward substitution.

S1.2 Define a matrix and column vector.

S1.3 Express a linear system in matrix-vector format or as an augmented matrix.

S1.4 Perform matrix addition, multiplication, and scalar multiplication whenever appropriate by hand and in MATLAB.

S1.5 Read MATLAB code involving for-loops, if-statements, and while loops.

S1.6 Compute the flop count of simple MATLAB codes.

Unit 2. The LU & PLU Factorization

S2.1 Define and compute the three row operations on an augmented matrix of any size.

S2.2 Express a given row operation as an elementary matrix. Obtain the inverse of these elementary matrices.

S2.3 Perform regular Gaussian elimination on $n \times n+1$ augmented matrices, when possible, and use back substitution to solve the underlying linear system.

S2.4 Obtain the LU decomposition of square matrices, when possible. Use forward and backward substitution on the LU decomposition to solve the underlying linear system.

S2.5 Obtain the inverse of a square matrix by Gaussian elimination, possibly allowing pivoting.

Unit 3. The QR Factorization

S3.1 State the canonical basis vectors of $\mathbb{R}^n$.

S3.2 Interpret column vectors geometrically as elements ("arrows") in the vector space $\mathbb{R}^n$.

S3.3 Interpret linear systems as finding the correct linear combination of a set of column vectors that decomposes a given column vector (i.e. the "column picture" of a linear system).

S3.4 Define the span of a set of column vectors in $\mathbb{R}^n$. Define a subspace in $\mathbb{R}^n$.

S3.5 Define linear independence of a set of column vectors. Test whether a set of column vectors is linear independent using the row echelon form of a matrix. Define the rank/nullity of a matrix in terms of its number of linearly independent/linearly dependent column vectors.

S3.6 Define a basis of a subspace. Obtain a basis for a given span of vectors in $\mathbb{R}^n$.

S3.7 Define the transpose of a column vector and matrix. Express the dot product of two column vectors using the transpose.

S3.8 Use the dot product to compute angles between two vectors in $\mathbb{R}^n$ as well as compute the Euclidean norm of vectors.

S3.9 Determine if two column vectors are orthogonal to each other, define an orthonormal basis of a subspace of $\mathbb{R}^n$, and define an orthogonal matrix.

S3.10 Obtain an orthonormal basis of a subspace given a set of column vectors that span that space using the Gram-Schmidt orthogonalization procedure.

S3.11 Obtain the QR factorization of a matrix using the Gram-Schmidt orthogonalization procedure.

S3.12 Use the QR factorization of a full rank matrix to solve least squares problems.

Unit 4. The Spectral Decomposition

S4.1 Define a linear transformation and determine what a linear transformation does algebraically to a vector given what it does to basis vectors in its domain.

S4.2 Provide geometric examples of linear transformations in $\mathbb{R}^2$ using their canonical matrix representations.

S4.3 Know that matrix multiplication is a composition of linear transformations.

S4.4 Define the range and nullspace of a canonical matrix representation of a linear transformation, find a basis for these subspaces, and obtain dimensions for these subspaces.

S4.5 Determine when a square matrix that represents a linear transformation is invertible using the determinant of a matrix.

S4.6 Represent a matrix in a different coordinate system using the change-of-basis formula.

S4.7 Define and compute eigenvalues and eigenvectors of a square matrix.

S4.8 Obtain the spectral decomposition of square matrices that are diagonalizable.

Unit 5. The Singular Value Decomposition

S5.1 Define symmetric, semi-positive definite matrices, and know that, for any rectangular matrix A, $A^T A$ and AA^T are symmetric, semi-positive definite matrices.

S5.2 Define the singular value decomposition of any rectangular matrix. Anticipate the sizes of the U, Σ, and V^T matrices given any rectangular matrix.

S5.3 Obtain the singular values and left/right singular vectors of any rectangular matrix.

S5.4 Use the singular value decomposition to express the best low-rank approximations of any rectangular matrix.

S5.5 Use the singular value decomposition to solve least squares problems.

S5.6 State the fundamental theorem of linear algebra and its connection to the singular value decomposition.

2 Holistic Course Goals

These are broader takeaways of the course designed to develop positive growth mindset in mathematics and to develop the student as a lifelong learner. The following list of goals will be accomplished through thoughtful reflections of homework assignments, quizzes, and in-class activities:

1. Use correct mathematical terminology in complete sentences.
2. Write up neat and organized homeworks, possibly using LaTeX.
3. Meet deadlines for assignments.
4. Cooperate with peers in an active learning environment.
5. Articulate their clearest and muddiest points in complete sentences.
6. Synthesize component skills listed above to solve multi-step or MATLAB problems.
7. Compare and evaluate different approaches to the same problem.
8. Explore problems that may seem unfamiliar, but follow from examples in class.
9. Appreciate that some problems in math, and in life, require more time and effort than others.
10. See mistakes as an opportunity to grow, not as a permanent setback.