Thm. (Arithmetic Mean - Geometric Mean)

If $a, b \geq 0$, then

$$\frac{a+b}{2} \geq \sqrt{ab}$$

Ex. $a=2, b=2$,

$$\frac{2+2}{2} = 2 \geq \sqrt{2 \times 2}$$

$a=1, b=9$,

$$\frac{1+9}{2} = 5 \geq \sqrt{1 \times 9}$$

If $a/b = 2$,

$$a/b = 2 \Rightarrow a = 2b$$

$$(a+b)^2 = 4ab$$

$a^2 - 2ab + b^2 = 0$, $(a-b)^2 = 0$

Proof. We know that $(a-b)^2 \geq 0$, or

$$a^2 - 2ab + b^2 \geq 0$$. By adding $4ab$ to both sides, it implies $a^2 + 2ab + b^2 = 4ab$, or

$$(a+b)^2 = 4ab$$. Since $a, b \geq 0$, we can take the square root of both sides to get $a+b \geq 2\sqrt{ab}$.

**Cases. If $P$ then $Q$**

- If $P$ then $P$ or $P_e$
- If $P$ then $Q$
- If $P_e$ then $Q$

Thm. If $a^2 \geq b$, then either $a \geq b$ or $b \geq a$.

$a=17, b=17$,

$a=9, b=9$

Proof. Use cases. Either $a \geq b$ or $b \geq a$.

Case 1. $a \geq b$. done.

Case 2. $a < b$ $\Rightarrow a^2 < b^2$, because $a/b < 1$.

Theorem. In every group of 6 people, there are either 3 mutual friends or 3 mutual strangers.

Proof. Use cases. Pick person $x$, either $x$ has at least 3 friends or at most 2.

Case 1: All of $x$'s friends are mutual strangers.

Case 2: $x$ has exactly 5 friends.

Case 2a: 3 mutual friends

Case 2b: $x$ has exactly 4 friends.

Case 2c: $x$ has no friends.

Theorem. If $r = 0$ and is irrational, so is $\sqrt{r}$.

Proof. Use contraposition.

If $\sqrt{r}$ is rational, then $\sqrt{r} = \frac{m}{n}$ for integers $m, n$, where $m$ and $n$ have no both integers so $\sqrt{r}$ is rational.

Converse: If $Q$ then $P$ is not always true.

If $0.5 \times 2$, then $0.5 \times (4 \times 0.5) = 0$ partial products converge is not true $x \approx 0.001$.

If $x^2 + 4x \geq 0$, then $0.5 \times 2$