Comp Sci 212

1. Stable Matching

Amendments
Final June 11 (info on canvas)

Stable matching - jobs applicants (equal #)
\[ \text{match} \text{ jobs} \leftrightarrow \text{ applicants} \]
(proposed preferences)

Def. Rogue couple - a job applicant pair
where both job applicant pair each other
over their current assignment.

Def. Stable matching - a perfect matching
no rogue couples.

Thm. There always exists a stable matching
(bipartite case)

Real (Gale-Shapley algorithm)

1. Each applicant interviews at their favorite job.
2. If job has too many applicants, reject all
   but their favorite.
3. If an applicant gets rejected, remove the job
   from their list.
4. Repeat 1-3 until every job has at most
   one applicant.

Possibly - Day 1, Shivas interview at Facebook
Day 2, Shivas and Timbu interview at Facebook
(Timbu get rejected on
day 1 from his favorite), Facebook
rejects Shivas.

P: for each job j and applicant a, if j is
   crossed off a's list, j prefers another
   applicant b to a, and j is b's favorite
   (not crossed off).

2. Assume that a does not get a job.
   Then must a job j that is unfulfilled.
   j rejects a, which by P implies that
   j has an interested applicant, contradicts
   j is unfulfilled.

3. Assume job j and applicant a are rogue.
   Either j is crossed off a's list, or a is
   Crook. If j is crossed off a's list,
   j prefers another interested applicant P,
   so j is can't be rogue.

Case 2: j's is crossed off a's list, a prefers
its job to j, so a & j can't be rogue.

Def. matching of a to j is possible if there
exists a stable matching matching a&j.

Thm. Each job gets at least one possible
applicant.

Proof: Assume j is matched with a, but
a is not an optimal possible applicant.
By preference j is a's optimal possible
job.

If a&j are matched, then a&j
are a rogue couple, so a is not possible
for j.

□
Sure, contradicts are a possible match.