Mathematical Tools for Neuroscience
J-Term (Jan 6-31) 2020, M/W/F 9:30-11:30am, GB122
Office hours: right after class

Instructors

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Schedule

Dates: MWF, Jan 6-31, 2020 *(10 classes total)*
Time: 9:30-11:30am
Room: GB122

Course Description and Prerequisites

Numerical data analysis has become a nearly indispensable tool in modern neuroscience. This course aims to equip graduate students with the fundamental mathematical skills in quantitative modeling and data analysis necessary for neuroscience research. The course is aimed at first or second-year students in the Neuroscience PhD program, and is open to other graduate students in the biosciences. This pilot course serves as a crash course to the basics of linear algebra, differential equations, and basic probability and statistics from a mathematical perspective. Each mathematical concept will be illustrated via applications to neural datasets.

Our goal is to make this as fun, approachable, and applicable as possible. We would like to build *mathematical intuition* for these essential topics. You will not need any math experience beyond high school calculus. Some amount of coding is necessary for this class; if you are rusty, this will be a chance to brush up your Matlab skills.

One goal in formulating this course was to alleviate the need for taking multiple undergraduate-level courses in each of the stated topics (which may be cumbersome due to back-and-forth commute, inconvenient scheduling, or just an excess of material with no clear applicability to neuroscience research).

Learning Objectives

By the end of this course, you will be able to:

- Build intuition for basic topics in linear algebra, matrix analysis, probability theory, dynamical systems, and differential equations.
- Recognize and apply appropriate quantitative modeling techniques across a range of common scenarios in neuroscience.
- Read and understand papers that contain these concepts.
- Explain essential concepts and methods in data analysis to scientific colleagues.

Class Materials

- Instructors will create and provide course notes. Lecture notes and homework will be posted on Canvas. We will post lecture notes by the end of the day of each lecture.

Grading

This course is graded SAT/UNSAT. We want to use grades as a motivator and not as a negative stressor. However, the only way to get good at mathematical intuition is through practice! We expect students to punctually submit complete homework assignments and to attend every class (unless prior arrangements have been made with the instructors). If you do these things, you will pass!

Expect to spend about 3 hours per week on homework. If you are spending more than that, please let us know.

Homework details:
- Homework is the single best way to practice the concepts we'll introduce in class.
- You should start on it early and engage in class with questions.
- There will be 3 problem sets, each covering approx. one week of class material. Submit problem sets via Canvas by 11:59pm on the due date.
- Please write out solutions with explanation so we can follow your logic.
- For problems requiring a mathematical response, please make sure you include work to show how you reached the solution. A "SAT" grade requires completion of all HW assignments, including all work on homework being shown.
- For problems requiring you to use a programming language, include the relevant figures and discussion in your writeup (i.e. embed the graphs in your text). It is not necessary to submit code (e.g. a .py or .m file) unless explicitly requested by the problem.
- We accept both typeset and handwritten solutions. If you choose the latter option, please make sure your handwriting is clear and that the scan is legible; we cannot give credit to solutions we cannot read! If you typeset your solutions, we strongly recommend LaTeX (overleaf.com)
- You are encouraged to work with others and consult external resources (books, internet, etc.) to do the assignments. In all cases, you must submit your own work.
- If you have extenuating circumstances, we will most likely grant you a 1-2 day homework extension if you email us in advance of the due date.

How is this course different from…?

You may be wondering how this course differs from the other quantitative offerings to select from.
- The ***MATLAB Bootcamp*** course that is required for all incoming G1's focuses primarily on how to use the MATLAB programming language, and is generally not concerned with mathematics.
- Rick Born's ***Thinking About Data*** course is an excellent course that address data analysis from a very practical perspective, and will allow you to explore the MATLAB functions necessary to implement statistical tests (t-tests), PCA, GLM fitting, bootstrap, etc. Rick's course will be very important for developing intuition by way of *simulation.* However, our course differs in two important ways: (1) we will aim to build mathematical intuition, and thus most of our practical exercises will be focused on solving problems in linear algebra, differential equations, and dynamical systems instead of coding; (2) we will not focus on frequentist statistical analyses (e.g. t-tests, chi-squared, etc.) for hypothesis testing, but will instead be assuming models (e.g. generalized linear models, probabilistic models) for the analyses of neural *population* data, i.e. we want to understand the dynamic structure of a collection of neurons.
- Jan's ***Probabilistic Modeling for Neural Data*** course *(new course, Spring 2020!)* is another excellent course that will focus on probabilistic modeling concepts that are commonly used to model neural data. This course will be mainly a paper-based methods course, exploring the different probabilistic models that have been used to fit and explain neural datasets. While Jan's course seems to address many similar concepts (such as Bayesian inference, GLMs, and dimensionality reduction), our course will be important for laying the foundational mathematical concepts that are already assumed before one take's Jan's course. It is important to note that Jan's course requires prerequisite knowledge of linear algebra and probability theory, both of which we will gain mathematical intuition for in our course, such that you have the necessary "vocabulary" to take more advanced courses (such as Jan's) in the future.

In conclusion, this course will be highly *complementary* to any other quantitative course that you choose to take in this department. If gaining fluency in computational skills is a goal of yours, it is generally a good idea to explore multiple courses during your time as a PhD student!

<u>Schedule</u>

**Note this schedule is subject to change!

Week	Date	Main Objectives/Highlights	Assignments/Reminders/Paper	Who?

1	**1-6 (M)**	*Lecture 1*
		• Vectors
		  ○ Vector spaces
		  ○ Geometric intuition
		  ○ Dot product, angle, similarity
		• Matrices
		  ○ Coordinates
		  ○ Change of basis
				ABC

	1-8 (W)	*Lecture 2*
		• Matrix decomposition
		  ○ Eigenstuffs
		  ○ SVD
		• Least squares regression
				LL

	1-10 (F)	*Lecture 3* • PCA • Applications of linear algebra concepts (e.g. dimensionality reduction techniques for population data)		LL
2	**1-13 (M)**	*Review/Overflow* • Overflow (if needed) • Review of basic linalg principles • Homework/question time		ABC/LL

	1-15 (W)	*Lecture 4*
		• Basic probability theory
		• Information theory
		  ○ Mutual information
		  ○ Entropy
			HW 1 due (linear algebra)	ABC

	1-17 (F)	PiN Interview Day		
3	**1-20 (M)**	Holiday (MLK)		
	1-22 (W)	*Lecture 5* • Differential equations • Dynamical systems		Guest lecture: John Assad
	1-24 (F)	*Lecture 6*		ABC

		• Application of dynamics to neural circuits (e.g. ring attractor)		

4	**1-27 (M)**	*Lecture 7*
		• Estimation and inference
		    ○ Bayes' rule
		    ○ Neural encoding models
		    ○ Maximum likelihood estimation
		    ○ Bayesian estimation
			HW 2 due (information theory and dynamics)	LL

	1-29 (W)	*Lecture 8* • General and generalized linear models		LL
	1-31 (F)	*Review/Overflow* • Overflow (if needed) • Review/question time • Course wrap-up and discussion	HW 3 due by 2-3 (estimation/inference and GLMs)	ABC/LL