Physical Sciences 2: Assignment for Sept 12 - Sept 19
Homework #2: Forces, Impulse, and Kinematics
Due Thursday September 19, at 9:00AM

This assignment must be turned in by 9:00AM on Thursday September 19. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to the mailbox marked with your section TF’s name in the row of mailboxes outside of Sci Ctr 108.

You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

After completing this homework, you should…

• Be able to use both definitions of impulse interchangeably
• Be able to calculate impulse from a graph of force vs. time
• Know Newton’s second and third laws
• Use dimensional analysis to identify proper units for an unknown quantity
• Use dimensional analysis to derive forms of equations
• Know the kinematic equations and when/how to use them
• Be able to interpret graphs of position vs. time, velocity vs. time, and accelerations vs. time
• Understand the conditions on acceleration of an object during free fall and during projectile motion
Here are summaries of this lecture’s important concepts to help you complete this homework:

**Impulse**
- When two objects collide, they experience a force over a period of time that changes the momentum.
- Impulse $J$ is the change in momentum:
  $$ J = \Delta \mathbf{p} = \int_{t_i}^{t_f} \mathbf{F}_{\text{net}} \, dt = \langle \mathbf{F}_{\text{net}} \rangle \Delta t $$
- On a force vs. time curve, impulse is the area under the curve.

**Kinematics**

**Interpreting Graphs**
- Position vs. time
  $$ \text{slope of } s \text{ vs. } t \text{ gives } v $$
- Velocity vs. time
  $$ \text{slope of } v \text{ vs. } t \text{ gives } a $$
  $$ \text{area under curve gives the change in position } \Delta s $$
- Acceleration vs. time
  $$ \text{area under curve gives the change in velocity } \Delta v $$
Kinematics

In the special case of constant acceleration, the following equations relate an object’s position, velocity, and acceleration at any time:

\[
\begin{align*}
\vec{V} &= \vec{V}_0 + \vec{a} \cdot t \\
\vec{r} &= \vec{r}_0 + \vec{V}_0 \cdot t + \frac{1}{2} \vec{a} \cdot t^2 \\
\vec{v}^2 &= \vec{v}_0^2 + 2 \vec{a} \cdot (\vec{r} - \vec{r}_0)
\end{align*}
\]

In the case of *free fall*, the only acceleration is due to gravity, so

\[
\begin{align*}
a_x &= 0 \\
a_y &= -g = -9.81 \text{ m/s}^2
\end{align*}
\]

- This means that an object's velocity in the x-direction will not change
  \[v_x = \text{constant}\]

Typical questions:

- For an object fired with initial velocity \(\vec{V}_0\) at an angle \(\theta\) to the ground,
  - How high does the object rise?
  - How much time passes before the object reaches the highest point of its trajectory?
  - How much time is the object in the air?
  - What is the total horizontal distance traveled by the object?

All of these questions can be answered by using some combination of the kinematic equations.
0. Reflections on Last Assignment

Pick one question from Homework 1 that you found particularly difficult and:

a) describe the errors that you made

b) ways to ensure that you have learned from your mistakes so that you won’t have the same trouble with such a question in the future

1. Projectile Motion Tutorial (2 pts)

A projectile is fired at an angle $\theta$ from the ground at an initial velocity of $v_0$.

a) What is the velocity of the projectile in the x-direction? Express your answer in terms of $\theta$ and $v_0$ as well as any fundamental constants.

b) What is the velocity of the projectile in the y-direction? Express your answer in terms of $\theta$ and $v_0$ as well as any fundamental constants.

c) How long does it take until the projectile has reached the highest point in its trajectory? Express your answer in terms of $\theta$ and $v_0$ as well as any fundamental constants.

d) How long does it take until the projectile reaches the ground? Express your answer in terms of $\theta$ and $v_0$ as well as any fundamental constants.

e) How far does the projectile travel? Express your answer in terms of $\theta$ and $v_0$ as well as any fundamental constants.

2. Putting Everything Together (Exam-Type Question): Adam and Betty (1 pt)

Adam and Betty are studying physics together:

Adam: “I think Newton’s 3rd law is wrong.”
Betty: “…ummm….why?”
Adam: “Well, think about someone walking. I know there’s an equal and opposite force between the ground and me. That makes the net force zero, so I shouldn’t go anywhere, but I clearly am able to walk places.”
Betty: “Ahhh, actually Newton’s still right because….”

What flaw in Adam’s reasoning did Betty find? (Maximum 2-3 sentences.)
3. Putting Everything Together: It’s a drag (2 pts)

There are two different “laws” for the drag force on an object moving through a fluid such as air or water. Using dimensional arguments, we can deduce the form of these two laws, but not which law is valid in any given situation.

In both laws, the magnitude of the drag force can depend on the size of the object (ℓ, a length) and on the speed (v, the magnitude of the velocity) of the object.

a) One of the expressions for drag force depends on the viscosity of the fluid (denoted by η, the Greek letter eta). Viscosity has SI units of kg·m⁻¹·s⁻¹. Determine the expression for the magnitude of the drag force that depends on the viscosity of the fluid. Your answer should be in the form of a proportionality, i.e. \( F_{\text{drag}} \propto \ell^a v^b \eta^c \) where the exponents \( a \), \( b \), and \( c \) are chosen so that the dimensions work out correctly.

b) The other expression for drag force depends on the density of the fluid (denoted by \( \rho \), the Greek letter rho), but not on the viscosity. Density has SI units of kg·m⁻³. Determine the expression for the magnitude of the drag force that depends on the density of the fluid; again, your answer should be in the form of a proportionality.

The following graph shows the horizontal component of the velocity as a function of time of the chest of a crash-test dummy sitting in the front passenger seat of a car during a 35-mph frontal impact. The four curves on the graph are (from left to right) (i) the velocity of the dashboard, (ii) the velocity of a dummy wearing a seatbelt with special safety devices (pretensioners), (iii) the velocity of a dummy wearing a seatbelt with no special safety devices, and (iv) the velocity of a dummy wearing no seatbelt.

![Graph showing velocity history of a dummy](image)


a) Assuming the dummy has a mass of 70 kg, use the graph to estimate the magnitude of the maximum force on the dummy during the collision for each of the three possibilities: the safety-seatbelt, the ordinary seatbelt, and no seatbelt.

b) Serious tissue injury occurs if the impact force per unit area exceeds about $5 \times 10^5 \text{ N} \cdot \text{m}^{-2}$. Assuming that the forces in a) are distributed over an area of about 500 cm$^2$, comment on the severity of the injuries that would likely be sustained in each of the three possible types of impact.
5. Putting Everything Together: Shoot the monkey (2 pts).

In lecture we saw the “Shoot the Monkey” demo. In short a spring-loaded gun is placed on a platform and aimed at a monkey a horizontal distance $D$ away and a vertical distance $h$ off the ground (see figure). Assuming the monkey doesn’t hit the ground first, you will show (by solving the below questions) that it will always get hit. Note: you will need to define some parameters that aren’t given in the drawing to solve this problem.

![Diagram of shooting monkey](image)

a) What is the time it takes the bullet to go a horizontal distance $D$.

b) What is the height of the bullet at that time? What is the height of the monkey at that time? Are they the same?

6. Firing a Mortar Downhill (2 pts)

NOTE: VIDEO TUTORIAL OF THIS PROBLEM ON CANVAS

A mortar crew is positioned near the top of a steep hill. Enemy forces are charging up the hill and it is necessary for the crew to spring into action. Angling the mortar at an angle of $\theta = 52.0^\circ$ (as shown), the crew fires the shell at a muzzle velocity of 258 feet per second.

a) How far down the hill does the shell strike if the hill subtends an angle $\phi = 37.0^\circ$ from the horizontal? (Ignore air friction.)

b) How long will the mortar shell remain in the air?

c) How fast will the shell be traveling when it hits the ground?