Newton's Second Law

\[ F_{\text{net}} = ma \]

- **Problem-Solving Strategy**:
  1. Draw a picture; draw and label all forces present.
  2. Choose a coordinate system.
  3. Write Newton's 2nd law in each direction.
  4. Solve for unknown quantities.
  5. Plug in numbers last.

**Forces**

- **Gravitational Force** $F_g$
  - Force due to gravity; equal to an object's mass multiplied by the gravitational acceleration $g = 9.81 \text{ m/s}^2$
  \[ F_g = mg \]

- **Normal Force** $N$
  - Contact force that always acts perpendicular to a surface.
  - Magnitude equal to whatever is necessary in order to keep the object from falling through the surface.

- **Tension Force** $T$
  - Pulling force that acts in the direction of the string and is the same at every point on the string.
**Forces**

- **Friction**
  - Static Friction $f_s$: friction force that resists the motion of two bodies at rest w.r.t. each other
    
    maximum value: $f_{s,\text{max}} = \mu_s N$

  - Kinetic Friction $f_k$: friction force that resists the motion of two bodies w.r.t. each other
    
    $f_k = \mu_k N$

- **Drag**
  - Viscous Drag: drag force for small objects moving slowly through viscous fluids
    
    $F_{\text{drag}} = 6\pi \eta RV$
    
    $\eta$ - viscosity of fluid
    $R$ - radius
    $V$ - speed
  
  - Pressure Drag: drag force for large objects moving quickly through non-viscous fluids
    
    $F_{\text{drag}} = \frac{1}{2} C_d \rho A V^2$
    
    $\rho$ - density of fluid
    $A$ - cross-sectional area
    $V$ - speed
    $C_d$ - drag coefficient
1. Two blocks 1 and 2, on a frictionless table, are pushed from the left by a horizontal force \( F_1 \) and on the right by a horizontal force \( F_2 \), as pictured. The magnitudes of the pushing forces satisfy the inequality

\[ |F_1| > |F_2| \]

Which of the following statements is true about the magnitude of the contact force \( N \) between the two blocks?

A) \( N > |F_1| > |F_2| \)

B) \( |F_1| > N > |F_2| \)

C) \( |F_1| > N = |F_2| \)

D) \( |F_1| = N > |F_2| \)

E) \( |F_1| = |F_2| > N \)

→ because \( |F_1| > |F_2| \), the two blocks will move to the right

→ By Newton's 3rd law, the force of block 1 on block 2 is equal to the force of block 2 on block 1. This is the "contact force" between the two blocks.
2. You push on a block of mass $M$ with a horizontal force $F$, as shown below. A block of mass $m$ on top of $M$ moves precisely along with it. What force directly causes $m$ to accelerate horizontally along with $M$?

![Diagram](image)

A) The normal force between the blocks  
B) The static friction force between the blocks

c) The kinetic friction force between the blocks  
D) The gravitational force on $m$

e) The force you apply on $M$  
f) No force is required because the masses are in contact

→ the static friction force keeps $m$ from losing contact with $M$ and directly causes $m$ to accelerate horizontally along with $M$.

→ the normal and gravitational forces act in the vertical direction; the kinetic friction force would appear if $m$ was moving with respect to $M$; the force you apply on $M$ does not directly affect $m$

What is the maximum magnitude of $F$ in order for $m$ to move along with $M$? How would kinetic friction on the surface under $M$ change this?

→ first, let's draw a free body diagram and write Newton's second law for $m$:

![Diagram](image)

$x$-direction: $F_{net, x} = ma$

$F_s = ma$

$y$-direction: $F_{net, y} = 0$

$N_m - mg = 0$

$N_m = mg$

\[
\text{NOTE: } F_s \text{ points to the right since, without it, } m \text{ would move to the left. The friction force acts in the direction opposite of this movement.}
\]
because we are trying to solve for the maximum magnitude of $F$, the friction force $F_s$ will also be at its max value:

$$F_s = \mu mg$$

the two blocks are moving together, so they move at the same acceleration, which can be solved for by

$$F_s = ma$$

$$\mu mg = ma$$

$$a = \mu g$$

now let's draw a free body diagram and write Newton's second law for $M$:

\[ \begin{align*}
F &= N_m \\
Mg &= 0 \\
N_m &= Mg + N_m \\
N_m &= Mg + mg \\
N_m &= (M+m)g
\end{align*} \]

NOTE: Block $M$ has two normal forces: $N_m$ due to the ground and $N_m$ due to block $m$.

Also, $F_s$ now points to the left since block $M$ is moving to the right.

we have already solved for $F_s$ and $a$, so we can now determine the maximum magnitude of $F$:

$$F - F_s = Ma$$

$$F = F_s + Ma$$

$$= (\mu mg) + M(\mu g)$$

$$F = (M+m)\mu g$$

Maximum magnitude, frictionless table.
2. Now, if the table was not frictionless, a kinetic friction force \( f_k \) would also be applied to block \( M; \) in this case, the free body diagram and Newton's second law become

**X-direction**

\[
F_{net,x} = Ma
\]

\[
F - f_s - f_k = Ma
\]

\[
F = Ma + f_s + f_k
\]

**Y-direction**

\[
F_{net,y} = 0
\]

\[
N_M - N_m - Mg = 0
\]

\[
N_M = N_m + Mg
\]

\[
N_m = (M+m)g
\]

\[
f_k = \mu_k N_m
\]

\[
f_k = \mu_k (M+m)g
\]

→ the acceleration and static friction force remain unchanged; the kinetic friction force is equal to

\[
f_k = \mu_k N_m
\]

\[
f_k = \mu_k (M+m)g
\]

→ we can now solve for the maximum magnitude of \( F \) in the presence of kinetic friction

\[
F = Ma + f_s + f_k
\]

\[
= M(M_s + \mu_s mg) + \mu_s mg + \mu_k (M+m)g
\]

\[
= \mu_s (M+m)g + \mu_k (M+m)g
\]

\[
F = (\mu_s + \mu_k)(M+m)g
\]

Maximum magnitude, kinetic friction present

→ As, the maximum magnitude of \( F \) increases in the presence of kinetic friction
3. If a 70-kg skier is subjected to a pressure drag force \( F_{\text{drag}} = \frac{1}{2} C_d \rho S A V^2 \), with \( C_d = 0.5 \), \( \rho = 1.2 \text{ kg/m}^3 \), and \( A = 0.5 \text{ m}^2 \), and is also subjected to a kinetic friction force with \( \mu_k = 0.1 \), calculate the terminal velocity for the skier on a slope that is inclined at \( \theta = 30^\circ \) relative to the horizontal.

For problems involving an inclined plane, it is most convenient to use a tilted coordinate system that is aligned with the plane.

We can solve for the terminal velocity by drawing a free body diagram and writing Newton’s second law:

\[ x\text{-direction} \]
\[ mgsin\theta - f_k - F_{\text{drag}} = 0 \]
\[ F_{\text{drag}} = mgsin\theta - f_k \]
\[ \frac{1}{2} C_d \rho S A V^2 = mgsin\theta - \mu_k N \]

\[ y\text{-direction} \]
\[ N - mgcos\theta = 0 \]
\[ N = mgcos\theta \]

\[ V^2 = \frac{2 (mgsin\theta - \mu_k mgcos\theta)}{C_d \rho A} \]
\[ V = \sqrt{\frac{2mg (sin\theta - \mu_k cos\theta)}{C_d \rho A}} \]

\[ \frac{2 (70 \text{ kg})(9.81 \text{ m/s}^2)(sin(30) - (0.1)cos(30))}{(0.5)(1.2 \text{ kg/m}^3)(0.5 \text{ m}^2)} \]

\[ V = 43.5 \text{ m/s} \]