Com Sc 212
1. Probability
2. Conditional Probability

Assignments
Midterm October 19
Practice midterm? 55- can ignore

Probability

Def. Sample space = non-empty set S
we S, outcomes (elements)
E ∊ S, event (subset)

Ex. S = {1, 2, 3, 4, 5, 6}
S' = {Heads, Tails}

Def. Probability function = total function
Pr: S → ℝ
s, w ∈ S
Pr[w] ≥ 0 for w ∈ S
Pr[w₁] + Pr[w₂] = 1

Ex. S = {1, 2, 3, 4, 5, 6} (fair die)

<table>
<thead>
<tr>
<th>w</th>
<th>Pr[w]</th>
<th>w</th>
<th>Pr[w]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/6</td>
<td>4</td>
<td>1/6</td>
</tr>
<tr>
<td>2</td>
<td>1/6</td>
<td>5</td>
<td>1/6</td>
</tr>
<tr>
<td>3</td>
<td>1/6</td>
<td>6</td>
<td>1/6</td>
</tr>
</tbody>
</table>

If E ⊆ S, Pr[E] = Σ Pr[w]
w ∈ E

Ex E = {1, 2, 3}

Pr[E] = 1/6 + 1/6 + 1/6 = 1/2

CS 305, die 9
Ex. Uniform distribution - Pr[w] = 1/6 for all w
Pr[E] = |E|/11

S = {H, T}¹₀₀, uniform distribution

1. Flipping a coin 100 times
Pr[At least 75 heads] = \# ways to get at least 75 heads

1512²⁰₀

Pr[S, s] = 1/2
S has at least 75 heads, 25 heads or less

Shirley coin S = {H, T}¹₀₀ Pr[At least 75 heads]
Pr[All heads] = 1/2
Pr[All tails] = 1/2
Pr[Everything else] = 0

Probability rules

- Sum, if E₁,..., Eₙ are disjoint tease
Pr[∑ u:E] = ∑ Pr[E]

- Complement Pr[S \ E] = 1 - Pr[E]

- Difference Pr[A \ B] = Pr[A] - Pr[B \ A \ B]

- If A ∩ B, Pr[A] ≤ Pr[B]

- Inclusion-Exclusion
Pr[A ∪ B] = Pr[A] + Pr[B] - Pr[A \ B]

- Union bound
Pr[A ∪ B] ≤ Pr[A] + Pr[B]

- A ∩ B, A ∩ B = ∑ Pr[A]

- Then (Union bound) Pr[A ∪ B] ≤ Pr[A] + Pr[B]

Proof Pr[A ∪ B] = \sum Pr[A]
w∈A

is either in w or in B

B get count at least once
Pr[A] + Pr[B]

Conditional probability

Pr[A|B] = Pr[A ∩ B] / Pr[B]

Given that the sum is 4, what is the probability
that the 1st die (or die roll) is 1?

Bₖ is set of outcomes whose sum is k.

B₁ = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), ...
A₂ set of outcomes with 1st die roll = 2

Ex. S = \{1, 2, 3\}^2⁵ sequences of length 25
(1, 2, 3) (1, 3, 2) (2, 3, 1) (3, 1, 2) (3, 2, 1)...

(1, 2, 3) (1, 3, 2) (2, 3, 1) (3, 2, 1) (3, 1, 2) (2, 1, 3)
(1, 3, 1) (1, 1, 3) (3, 2, 3) (3, 3, 2) (3, 1, 3) (3, 3, 1)
Example: \( S = \{100\}^{25} \) sequences of length 25

- Uniform dist. 100-sided die 25 times.

- \( P(LA) \) : probability of the roll is \( i \); \( P(LA) = P(A \cup A_1 \cup A_2 \cdots A_{25}) = \sum_{i=1}^{25} P(LA_i) \)

- \( A \) : event 1st roll is 1; \( A = \{ x \mid x \leq 5, x \neq 3 \} \)

- \( \vert A \vert = 100^{24}, \vert S \vert = 100^{25}, \text{ prob.} \ P(A) = \frac{1}{100} \)

- \( \text{prob. rule} \quad \text{prob. rule} \quad \)

- \( \text{prob} \ = \frac{1}{4} \quad \text{prob} \ = \frac{1}{4} \)

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\( P(A \cup B) = \frac{1}{3} \)

\( P(A) = \frac{1}{100}, \text{ prob. of event A} \)

\( P(B) = \frac{1}{4}, \text{ prob. of event B} \)

\( P(A \cap B) = \frac{1}{3} \)

\( P(A | B) = \frac{1}{3} \)

\( P(A) = \frac{1}{100}, \text{ prob. of event A} \)

\( P(B) = \frac{1}{4}, \text{ prob. of event B} \)

Given that a person has two kids, one is a boy, what is the probability that both are boys?

- \( S = \{(b, b), (b, g), (g, b), (g, g)\} \)

- Uniform dist.

- \( A \) : both are boys = \( \{(b, b)\} \)

- \( B \) : one is a boy = \( \{(b, b), (b, g), (g, b), (g, g)\} \)

\[ P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \]