Solutions to the activity

Question 1: Many pairs work, including this one
Question 2: All solutions must involve the "P" (purple below), or the cross (yellow below)! Only four solutions.
Solutions to the activity

Question 3: There is only one solution!
Solutions to the activity

Question 4: There is only one solution!
Solutions to the activity

Question 5: There is only one solution (up to internal rearrangement within each tiling)
Solutions to the activity

Question 6: There are five solutions total to make 2 shapes (it is not possible to make a third). Here is a solution:
Solutions to the activity

Question 7: There are 11 solutions total to make 2 shapes (it is not possible to make a third). Here is a solution:
Solutions to the activity

Question 8: Only one solution!
Solutions to the activity

Question 9: only 2 solutions for $3 \times 20$ (up to symmetry), but 2339 solutions for $6 \times 10$. 
Questions about tilings: Given a fixed set of tiles, can we tile a fixed shape with it...

► ... allowing the tiles to be translated only?
► ... allowing the tiles to be translated or rotated?
► ... allowing the tiles to be translated, rotated or flipped?
► ... allowing multiple copies of the tiles?
► ... with a single tile, but an unlimited number of copies?
► ... tiling a bounded region?
► ... tiling the whole plane?

If there exists a tiling with these constraints, is the solution unique? How many solutions are there?
Tiling a fixed area

Can you tile an $8 \times 8$ chessboard with dominoes if corners are missing?
Tiling a fixed area

Can you tile an $8 \times 8$ chessboard with dominoes if corners are missing?

No, because a domino covers a black and a white cell, and the number of white cells is not the same as the number of black cells.
Tiling a fixed area

What if the two tiles have the same color?
Tiling a fixed area

What if the two tiles have the same color?

It is always possible! Draw any cycle that connects all the cells. The length between the two red cells on this path is always even. Tile following this path.
Tiling a fixed area

What if we remove four tiles?
Tiling a fixed area

What if we remove four tiles?

It depends... The problem has several cases, so it is not as interesting.
Tiling a fixed area

Consider a $10 \times 10$ board. Is it possible to tile it with a $1 \times 4$ rectangle?
Tiling a fixed area

Consider a $10 \times 10$ board. Is it possible to tile it with a $1 \times 4$ rectangle?

No! A tile covers two tiles of each of two colors. There are 25 tiles of each.
Aztec diamond

What does a “typical” tiling looks like?
Artistic tilings

What if we don’t restrict ourselves to right angles?
Activity: In teams of 3-4, play with darts and kites, and try to build a tiling. What are your conclusions?
Artistic tilings

What if we don’t restrict ourselves to right angles?
Activity: In teams of 3-4, play with darts and kites, and try to build a tiling. What are your conclusions?
Tilings can either be periodic or aperiodic:
Some impressive aperiodic tilings are built using darts and kites, like the Penrose tiling:
Artistic tilings
Claim: The horse tile is “a square”!
Artistic tilings

Claim: The horse tile is “a square”!
Credits

- Julia Robinson Math Festival (for the activity)
- Roger Penrose
- M.C. Escher