Aliasing example

The signal \( \cos(2\pi ft) \) is sampled at \( \Delta T = 1 \) sample/s to produce the sampled signal \( \cos(2\pi fk) \) (that is, \( t = k\Delta T \), for \( k = -\infty, -1, 0, 1, 2, \ldots \)). What frequencies \( f \) are aliased to \( \cos(2\pi f_1k) \) for \( f_1 = 0.2 \) Hz?

\[
\cos(2\pi(f_1+n)k) = \cos(2\pi f_1 k) \quad \text{for} \quad n = -\infty, -1, 0, 1, 2, \ldots
\]

\textbf{and} \quad \cos(-2\pi(f_1+n)k) = \cos(2\pi f_1 k) \quad \text{for} \quad n = -\infty, -1, 0, 1, 2, \ldots

\Rightarrow f = \pm f_1 + n \quad \text{for} \quad n = -\infty, -1, 0, 1, 2, \ldots

\Rightarrow f = \pm 0.2 + n \quad \text{for} \quad n = -\infty, -1, 0, 1, 2, \ldots

\Rightarrow f = \pm f_1 + n/\Delta T \quad \text{in general}
Sampling Rate = 20 sample/sec
(Nyquist frequency = 10 Hz)
DAQ Assistant samples Analog Input Channel 0 each time this While Loop executes, i.e., every 1/20 sec.
The sampled versions of 1 Hz and 21 Hz sine waves (the red dots) are identical.

\[ \frac{2\pi}{\Delta T} = \frac{2\pi}{0.05} \text{ rad/sec} = 20 \text{ Hz} \]
What does this do?
Lab 3 low-pass filter

It implements the transfer function

\[ u(t) \rightarrow \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow y(t) \]

or, equivalently, it solves the corresponding differential equation

\[ \frac{d^2 y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y(t) = \omega_n^2 u(t) \]

for the output signal \( y(t) \) in real time.

On the breadboard, and electrical components from the lab 3, the low-pass filter diagrammed below.

For the circuit, choose:

\[ R_1 = 18,000 \, \Omega \quad R_2 = 30,000 \, \Omega \]
\[ C_1 = 1 \times 10^{-6} \, \text{F} \quad C_2 = 0.470 \times 10^{-6} \, \text{F} \]

Implement, using your myDAQ device and breadboard, and electrical components from the ME 473 small electronic parts cabinet in MEB 115, the low-pass filter diagrammed below.

For the resistance and capacitance values for your circuit, choose:

\[ R_1 = 18,000 \, \Omega \quad R_2 = 30,000 \, \Omega \]
\[ C_1 = 1 \times 10^{-6} \, \text{F} \quad C_2 = 0.470 \times 10^{-6} \, \text{F} \]

Use the LCR meter in MEB 115 to obtain the precise resistances of your two resistors and precise capacitances of your two capacitors.

What does this circuit do?

or, equivalently, it solves the corresponding differential equation for the output signal \( y(t) \) in real time.

\[ \frac{V_v(s)}{V_i(s)} = \frac{1}{\tau_1\tau_2 s^2 + (\tau_2 + \tau_3)s + 1} \]
\[ = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ \tau_1 = R_1C_1 \quad \tau_2 = R_2C_2 \quad \tau_3 = R_1C_2 \]

\[ \omega_n = \frac{1}{\sqrt{\tau_1\tau_2}} = \frac{1}{\sqrt{R_1C_1R_2C_2}} \]

\[ \zeta = \frac{\tau_2 + \tau_3}{2\sqrt{\tau_1\tau_2}} = \frac{(R_1 + R_2)C_2}{2\omega_n} \]
It implements a transfer function

\[ u(k) \rightarrow \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_0}{z^n + a_{n-1} z^{n-1} + \cdots + a_0} \rightarrow y(k) \]

\( n \geq m \)

or, equivalently, it solves the corresponding difference equation

\[ y(k) + a_{n-1} y(k-1) + \cdots + a_0 y(k-n) = b_m u(k-(n-m)) + b_{m-1} u(k-(n-m)-1) + \cdots + b_0 u(k-n) \]

for the output signal \( y(k) \) in real time.