Physical Sciences 2

Physical Sciences 2
Practice Final Exam

Your name: ___________SOLUTION__________________________________________

Section TF: ______________________________________________________________

Do not turn the page until you are told to begin. You will be given 3 hours to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper. Please box your final answer to each calculation.

You may use a calculator if you have brought one. You may refer to three 8.5”x11” sheets of notes, which must be in your own handwriting. Turn in your notes along with the exam itself when time is called.

This exam contains 10 sheets of paper (including this one), consisting of 9 problems. Do not write in the following table; it will be used for grading.

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Problem 1: Multiple Choice [35 points]

For each of the following questions, circle the letter of the best answer from the options given. Each question is worth 5 points. Partial credit will be given for those problems asking you to “Circle all that apply.”

a) Two pollen grains are undergoing Brownian motion in a container of water at temperature $T$. Pollen grain 1 has a mass that is twice that of pollen grain 2. How do their typical speeds compare to each other?

- A. They will have the same typical speed
- B. Pollen grain 2 will have a typical speed that’s $\sqrt{2}$ times the typical speed of pollen grain 1
- C. Pollen grain 2 will have a typical speed that’s twice that of pollen grain 1
- D. Pollen grain 2 will have a typical speed that’s four times that of pollen grain 1
- E. Pollen grain 1 will have a typical speed that’s $\sqrt{2}$ times the typical speed of pollen grain 2
- F. Pollen grain 1 will have a typical speed that’s twice that of pollen grain 2
- G. Pollen grain 1 will have a typical speed that’s four times that of pollen grain 2
- H. There is not enough information in the question

Here, both have the same average/typical kinetic energy: $3/2 k_B T$. So $1/2 m_1 v_1^2 = 1/2 m_2 v_2^2$. So, the less massive grain is moving faster by a factor of square root of 2

b) You drop a ball directly down off a tall bridge, but you put spin on the ball so that the top of the ball is moving to the left. When the ball hits the water below the bridge, where (horizontally) is the ball relative to you?

- A. Directly below you
- B. In front of you
- C. Behind you
- D. To the right of you
- E. To the left of you

If the top of the ball is moving left, then the left side of the ball is moving down and the right side of the ball is moving up. The wind moves up past the ball as it falls. On the right side of the ball the upward wind plus the upward motion of the ball makes the air move faster than on the left side where the upward motion of the ball counteracts the downward motion of the ball making the air move slower. Based on the Venturi effect, if there is a place of faster moving fluid (right side of ball), then it will have a low pressure compared to a place where the air is moving slower (left side of ball). So the high pressure on the left side of the ball and the low pressure on the right side of the ball cause the ball to the right.

4. Two identical spherical drops of water are falling through the air in free fall. Which of the following actions would decrease the total surface energy of these two drops of water? Circle all that apply.

- (a) Break up the two drops into many smaller drops.
- (b) Distort one of the drops into an elliptical (non-spherical) shape.
- (c) Increase the surface tension $\gamma$ of the water by adding a surfactant to both drops.
- (d) Combine the two drops into a single, larger spherical drop.
- (e) Combine the two drops to form a long, thin “tube” of water 1 meter long.
d. (5 points) A heavy box is initially at rest at the top of a ramp. A girl pushes the box down the ramp. At the bottom, she stops pushing and the box comes to a stop due to friction. Fill in the blanks: For the entire process described above, the work done by the girl on the box was _______ and the work done by gravity on the box was _______.

(a) positive; positive  
(b) positive; negative  
(c) positive; zero  
(d) zero; positive  
(e) zero; negative  
(f) zero; zero

---

e. (5 points) You suspend a weight from a 30 cm long wire; the wire stretches by 1 cm. You cut the wire in thirds. You discard one piece of wire and suspend the same weight from **two** of the 10 cm pieces of wire. How much does each 10 cm piece of wire stretch?

(a) 1/9 cm  
(b) 1/6 cm  
(c) 1/4 cm  
(d) 2/3 cm  
(e) 3/2 cm  
(f) 4 cm  
(g) 6 cm  
(h) 9 cm
f. (5 points) Which of the following situations describe an object with zero velocity, but a nonzero acceleration? (Choose all that apply; partial credit will be given.)

(a) A ball thrown straight up in the air at the highest point in its trajectory
(b) A pendulum at the middle of its swing
(c) A pendulum at the extremes of its swing
(d) A ball bouncing straight up off the ground, at the point where it is instantaneously at rest on the floor
(e) A book at rest on a table

---

f. (5 points) Two different size masses speeding towards each other collide and stick together after the collision. The system (composed of both masses only) was completely isolated. **Circle all that apply.**

(a) Momentum was conserved but mechanical energy was **not** conserved.
(b) The internal energy of the system **decreased** during the collision.
(c) The internal energy of the system **increased** during the collision.
(d) The kinetic energy of the system after the collision is equal to the **sum** of kinetic energies of both masses before the collision.
(e) During the collision, the two masses received **equal** and opposite (direction) impulses.
2. (21 points) You place an unknown mass \( m \) on a frictionless horizontal surface, and attach it to a wall with a spring as shown below. The spring has a spring constant \( k = 200 \text{ N/m} \). When the spring is relaxed at its equilibrium length, the mass is at position \( x = 0 \).

You stretch the spring, release it, and record the position of the mass as a function of time, as shown below. Please justify your answers to the following questions with a brief explanation or by evoking the appropriate mathematical relation. Each part is worth 3 points.

(a) What is the period, \( T \)?

\[ T = 0.4 \text{ s} \]

(b) What is the angular frequency, \( \omega \)?

\[
\omega = 2\pi f = \frac{2\pi}{0.4\text{ s}} = 5\pi \text{ rad/s}
\]

(c) What is the maximum speed of the mass during the oscillation, \( v_{\text{max}} \)?

\[
v_{\text{max}} = Aw = (0.03 \text{ m})(5\pi \text{ rad/s}) = 0.15\pi \text{ m/s}
\]
(d) Write an expression for the position of the oscillator as a function of time, \( x(t) = \ldots \) including all of the appropriate units. (Hint: At \( t = 0 \) the mass is at rest with \( x(0) = A = 3\text{cm} \).

\[
x(t) = 0.03m \cos\left(\frac{\pi \text{ rad}}{5} \cdot t\right)
\]

(e) For which values of \( x \) is the magnitude of the mass’s acceleration greatest?

+3cm and -3cm

(f) What is the total mechanical energy of the mass, in units of Joules, at these locations?

\[
E = \frac{1}{2} k A^2
\]

\[
= 0.09 \text{ J}
\]

(g) What is the change in the spring’s potential energy, in units of joules, as the mass goes from \( x_i = 3 \text{ cm} \) to \( x_f = 1 \text{ cm} \)?

\[
\Delta PE = \frac{1}{2} k (x_f^2 - x_i^2)
\]

\[
= \frac{1}{2} (200 \text{ N/m}) (0.01 \text{ m}^2 - 0.03 \text{ m}^2)
\]

\[
= -0.08 \text{ J}
\]
3. (20 points) Marvin's Mass

A uniform plank of mass \( M \) and length \( L \) is suspended from the ceiling by a pair of ropes positioned as shown. Marvin, mass \( m \) (the block below), rests on the plank at a distance \( x \) from the left end (represented by the grey square).

(a) (10 points) What is the minimum value of \( x \) for which the plank will not tip? Your answer may be expressed in terms of \( M \), \( m \), and \( L \).

This threshold is met at the instant \( T_2 \to 0 \)

Choose \( T_1 \) as pivot

\[
\Sigma T = \left( \frac{2}{3} L \right) T_2 - \left( \frac{1}{3} L \right) F_g + \left( \frac{1}{3} L - x \right) F_M = 0
\]

\[
\Rightarrow - \frac{L}{6} M g - \frac{1}{3} m g - m g x = 0
\]

\[
+ \frac{L}{3} \left( \frac{M}{2} - m \right) + m x = 0
\]

Check: if \( m = M \), should get \( x = \frac{L}{6} \)

\[
x = \frac{L}{3} \left( 1 - \frac{1}{2} \right) = \frac{L}{3} \left( \frac{1}{2} \right) = \frac{L}{6}
\]
(b) (10 points) If Marvin sits at a distance of $x = L/3$, right where the left cable is attached, what are the tensions in each of the two cables?

We can get $T_2$ from our torque eqn

$$\frac{2}{3}LT_2 - \frac{1}{6}LMg + (\frac{1}{3} - \frac{L}{3})F_m = 0$$

$$\frac{2}{3}T_2 = \frac{1}{6}YMg$$

$$T_2 = \frac{1}{4}YMg$$

$$\sum F_y = 0$$

$$T_1 + T_2 - (M+m)g = 0$$

$$T_1 = (M+m)g - \frac{1}{4}YMg$$

$$T_1 = (\frac{3}{4}M+m)g$$
4. (20 points) Marvin the tiny mammoth is roaming a glacier when he encounters a deep ravine that he cannot cross. Fortunately, the glacier is shaped as shown below. The bottom of the big hill is a circular segment of radius $R$. After Marvin takes a few careful measurements, he climbs the big hill to height $h$, and, starting from rest, slides down the frictionless hill and shoots across the ravine, just barely making it across. Marvin has mass $m$. You may neglect friction and drag in this problem.

(a) (10 Points) What is the magnitude and direction of the normal force when Marvin reaches the bottom of the jump (the point marked A) in the figure? Your answer may be expressed in terms of any of the variables given in the problem and any physical constants.

\[ \sum F_y = ma_y \]
\[ F_N - F_g = m\frac{v^2}{R} \]
\[ F_N = mg + \frac{mv^2}{R} \]

\[ mgh = \frac{1}{2}mv^2 \]
\[ v^2 = 2gh \]

\[ F_N = mg + 2mgh \frac{v^2}{R} \]
(b) (5 Points) What is Marvin’s speed when he first becomes airborne (point B in the figure)? Your answer may be expressed in terms of any of the variables given in the problem and any physical constants.

\[ mgh = \frac{1}{2}mv^2 + mgR \]

\[ v^2 = 2gh - 2gR \]

\[ v^2 = 2g(h - R) \]

\[ v = \sqrt{2g(h - R)} \]

(c) (5 Points) How high up the hill did Marvin start? Your answer may be expressed in terms of any of the variables given in the problem (except \( h \)) and any physical constants.

\[ D = v \cos \theta t \]

\[ t = \frac{D}{v \cos \theta} \]

\[ 0 = v \sin \theta t - \frac{1}{2}gt^2 \]

\[ v \sin \theta = \frac{1}{2}gt \]

\[ 0 = v \sin \theta = \frac{gD}{v \cos \theta} \]

\[ 2v \sin \theta \cos \theta = gD \]

\[ v^2 = \frac{gD}{2 \sin \theta \cos \theta} \]

\[ 2g(h - R) = \frac{gD}{2 \sin \theta \cos \theta} \]

\[ h = R + \frac{D}{4 \sin \theta \cos \theta} \]
5. (15 points) A 20.0 kg \((M)\) block is connected to an empty 3.0 kg \((m)\) bucket by a massless rope running over a frictionless, massless pulley, as shown below. The coefficient of static friction between the table and the block is 0.450 and the coefficient of kinetic friction between the table and the block is 0.320. Sand is slowly added to the bucket until the system begins to move. From the moment it begins to move, no more sand is added to the bucket.

(a) (5 Points) Draw free body diagrams for the block and for the bucket right before the system begins to move.

(b) (5 Points) Calculate the mass of sand added to the bucket.

\[ \text{When does } T = F_{\text{fs max}}? \]

\[ \mu F_N = mg \]

\[ \mu M g = mg \]

\[ m = m_{\text{bucket}} + m_{\text{sand}} \]

\[ m_{\text{bucket}} + m_{\text{sand}} = \mu M \]

\[ m_{\text{sand}} = \mu M - M_{\text{bucket}} \]
(c) (5 Points) How much time does it take the bucket to fall a distance of 1.0 meter from its original height?

We need to find the acceleration $\ddot{y}$ of the bucket to get the time, since $\Delta y = \frac{1}{2} at^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{a}}$

**FBD for bucket:**

- $\Sigma F_y = m_{\text{bucket}} a$
- $-T + F_{\text{bucket}} = m_{\text{bucket}} a$

**Use FBD for block:**

- $\Sigma F_x = T - F_w = m_{\text{block}} a_{\text{block}}$
- $T = m_{\text{block}} a_{\text{block}} + F_w$

$$-m_{\text{block}} a - \mu m_{\text{block}} g + m_{\text{bucket}} g = m_{\text{bucket}} a$$

$$\ (m_{\text{bucket}} - \mu m_{\text{block}})g = (m_{\text{bucket}} + m_{\text{block}})a$$

$$a = \left( \frac{m_{\text{bucket}} - \mu m_{\text{block}}}{m_{\text{bucket}} + m_{\text{block}}} \right) g$$

$$t = \sqrt{\frac{2\Delta y}{a}} = 1.5 s$$
Problem 6: Car Loop [28 points]

A toy car of mass m starts at the top of a ramp of height 2R, and then frictionless travels toward a circular loop of radius R.

a) Find expressions for both the normal force on the car when it reaches point A (where the track is flat) and point C.

b) Rank the magnitude of the size of the normal force on the car, when it is at points A, B, and C.
c) Find an expression for the normal force on the car at point D, which is an angle \( \theta \) from the horizontal, as shown in the figure.

d) At what maximum angle \( \theta \) will the car reach before it falls off the track?
Problem 7: Slow Pump  [26 points]

Below is a device that one could use to slowly pump water out of a tube, while simultaneously measuring the pressure (at a chosen point) in the tube by looking at the height of water in a connected capillary tube. It consists of a horizontal tube of length $2L$, a vertical tube protruding vertically from the center of the horizontal tube, a large reservoir of water, and above which is a cylinder of air which is initially at atmospheric pressure, when the piston (a.k.a. plunger) is $L = 1$ meter above the water’s surface. The radius of both horizontal and vertical tubes is very narrow, $1$ micrometer. Also, both tubes are open to the atmosphere. $\eta_{\text{water}} = 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}, \rho_{\text{water}} = 10^3 \frac{\text{kg}}{\text{m}^3}$

a) If you push the plunger down, compressing the air, so its height above the water’s surface is $L/9$, what is the flow rate out of the horizontal tube? Assume air temperature below piston is kept constant, and assume the water in the vertical tube is static.
b) What is the pressure at point $P_1$, the vertex of the two tubes?

\[ Q = \frac{\pi}{8} \frac{\Delta P R^4}{\gamma} \]

\[ \Rightarrow \Delta P = \frac{8 \gamma y l}{\pi R^4} \]

\[ \Rightarrow P_{\text{left side of tube}} - P_c = \frac{8 \gamma y l}{\pi R^4} \]

\[ \Rightarrow P_c = P_{\text{left side of tube}} - \frac{8 \gamma y l}{\pi R^4} \]

\[ \Rightarrow P_c = 9.1 \times 10^5 \text{Pa} - \frac{8 \times 1.6 \times 10^{-15} \text{m}^3}{\pi \times 10^{-4} \text{m}^4} \]

\[ = 9.1 \times 10^5 \text{Pa} - 4.1 \times 10^5 \text{Pa} \]

\[ \approx 5 \times 10^5 \text{Pa} \]

c) Assuming the water in the vertical tube remains static and a spherical meniscus of radius $R$ is formed at its surface, how high will the water level $\Delta h$ be? $\gamma_{\text{water}} = 0.073 \text{ N/m}$

\[ C.) \quad \text{Following the pressure change from above meniscus to the vertex of the two tubes, we can find:} \]

\[ P_c = \text{atm} - \frac{2 \gamma}{R} + \rho g \Delta h \]

\[ \Rightarrow \Delta h = \left( P_c - \text{atm} + \frac{2 \gamma}{R} \right) / \rho g \]

\[ \Rightarrow \Delta h = \left( 5 \times 10^5 \text{Pa} - 1.01 \times 10^5 \text{Pa} + \frac{2 \times 0.073 \text{ N/m}}{10^{-6} \text{ m}} \right) / \rho g \]

\[ \Rightarrow \Delta h \approx 55 \text{ m} \]
Problem 8: Cone-ing Down the Drain  [25 points]

At the bottom of a deep reservoir, water is being pumped down a (inverted) cone-shaped drain, as shown to the right. As the water enters the drain it is at a pressure of 10 atm and is moving down at a flow rate of 100 m$^3$/s. The cone-shaped drain has a radius of 11 meters at the top, a radius of 1 meter at the bottom, and 10 meters of height between the top and bottom. $\rho_{\text{water}} = 10^3 \frac{kg}{m^3}$

a) What is the speed of the water in the cone as a function the height, $y$. $y = 0$ at the bottom of the cone and $y = 10$ meters at the top of the cone. Express you answer in terms of numbers and “$y$” (include units for the numbers).
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b) What is the pressure of the water in the cone as a function the height, y. Express you answer in terms of numbers and “y” (include units for the numbers; it’s ok if the expression isn’t fully simplified).

We want to use Bernoulli: \( P_1 + \frac{1}{2} \rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g y_2 \).

Set point 2 at top of tank and point 1 at height y in tank.

- \( P_2 = 10 \text{ atm} = 1.01 \times 10^6 \text{ Pa} \)
- \( V_2 = \frac{100 \text{ m}^3}{\pi (1 \text{ m})^2} \)
- \( y_2 = 10 \text{ m} \)

\[
P_2 + \frac{1}{2} \rho V_2^2 + \rho g y_2 = 1.01 \times 10^6 + \frac{1}{2} \rho \frac{100}{\pi (1 \text{ m})^2}^2 + 10 \times 9.81 \times 10 \]

\[
\approx 1.1 \times 10^6 \text{ Pa}
\]

So Bernoulli becomes:

\[
P_1 + \frac{1}{2} \rho V_1^2 + \rho g y_1 = 1.1 \times 10^6 \text{ Pa}
\]

\[
\Rightarrow \quad P_1 = 1.1 \times 10^6 \text{ Pa} - \rho g y_1 - \frac{1}{2} \rho V_1^2(y)
\]

\[
\Rightarrow \quad P_1(y) = 1.1 \times 10^6 \text{ Pa} - \rho g y - \frac{1}{2} \rho \frac{100}{\pi (1 \text{ m} + y)^2}^2
\]

\[
\Rightarrow \quad P_1(y) = 1.1 \times 10^6 \text{ Pa} - 9.8 \times 10^5 \frac{\text{ kg}}{\text{ m}^2} \cdot y - \frac{5 \times 10^5 \frac{\text{ kg}}{\text{ m}^2}}{(1 \text{ m} + y)^4}
\]

c) Use your expression in part b, with y = 10 meters, and confirm that you get 10 atm.

Confirming that we get 10 atm @ 10m

\[
P_1(10\text{ m}) = 1.1 \times 10^6 \text{ Pa} - 9.8 \times 10^5 \text{ Pa} - 35 \text{ Pa}
\]

\[
\approx 1.0 \times 10^6 \text{ Pa}
\]

\[
\approx 10 \text{ atm}
\]
9. Consider potassium ions crossing a biological membrane 10 nm thick. $D$ for potassium in the membrane is $1.0 \times 10^{-16} \text{ m}^2/\text{s}$.

(a) **What number of potassium ions per second** will move across an area 100nm x 100nm if the concentration difference across the membrane is maintained at 0.50 mol/liter ($3 \times 10^{26}$ ions/m$^3$)?

(b) Discuss how the flux changes if we increase or decrease the membrane thickness.

Note: 1 litre = $10^{-3} \text{ m}^3$