Convolutional, Recurrent and Attention-Based Architectures

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Review: Deep Learning and Universality

- A system that employs a hierarchy of features of the input, learned end-to-end jointly with the predictor.

\[ f(x; \theta_1, \theta_2, \ldots, \theta_L) = F_L(F_{L-1}(\cdots F_2(F_1(x; \theta_1); \theta_2) \cdots); \theta_L) \]

- We will refer to \( F_k \) as layer \( k \)

- E.g., deep learning for classification:

\[ f_c(x; w, b, \theta_1, \theta_2, \ldots, \theta_L) = w_c \cdot f(x; \theta_1, \theta_2, \ldots, \theta_L) + b_c \]

- All parameters \((w, b, \theta_1, \theta_2, \ldots, \theta_L)\) are learned jointly

- We can think of \( f(x; \theta_1, \theta_2, \ldots, \theta_L) \) as learned features for \( x \) or a learned representation of \( x \) (doesn’t depend on the class being scored)

- **Universality.** Feedforward with a nonlinear layer can express any mapping (given enough hidden units).
Review: Computation Graph

Express any differentiable function as a directed acyclic graph (DAG) and automatically calculate gradients for all nodes.

- **Forward.** Populate values in topological order.

- **Backward.** Populate gradients in reverse topological order by the chain rule.

\[ z^i = \sum_{j \in \text{ch}(i)} z^j \times \frac{\partial f^j(x^j_I)}{\partial x^i} | x^j_I = a^j_I \]

Jacobian of \( f^j \) wrt. \( x^i \)

Use the stored gradients to update parameters.
Training Tips

- Regularization: Dropout, label smoothing, layer normalization
- Initialization: Uniform, normal, Xavier, Kaiming, and others
- Optimization: Appropriate learning rates, gradients with momentum, gradient clipping
- Ensembling: Average many stochastically trained neural models for variance reduction and improved generalization.
- More tricks:
  - Gradient accumulation: Make batch size $G$ times larger without using more memory by accumulating gradients over $G$ batches before updating weights.
  - Residual connection: Use $\text{enc}_\theta(x) + x$ to propagate gradient directly to $x$, useful with deep networks (hidden dim must equal input dim).
Review: Need for Specialized Neural Architectures

- Feedforward implicitly assumes the input is a single vector.
- NLP: Input is a *sequence*!
- Option 1: BOW representation
  - Loses lots of information (e.g., ordering), high-dimensional
- Option 2: Giant feedforward with input dimension $= \text{max sequence length}$
  - Computationally intractable, too many parameters to learn
- Solution: Develop specialized architectures that can handle *variable* input lengths.
- Starting point: *word embeddings*
  - Learn a dense low-dimensional vector for each word type.
Word Embeddings

\[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]
terrier

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
poodle

\[
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
frog

\shortmid \shortmid \text{poodle} - \text{terrier} \shortmid = \shortmid \text{terrier} - \text{frog} \shortmid = \shortmid \text{frog} - \text{poodle} \shortmid

\text{Generalization at word level!}
Embedding Matrix in Practice

- Part of model parameter $\theta$ to learn (aka. “lookup table”)
- $E \in \mathbb{R}^{d_w \times V}$ where $V = |\mathcal{V}|$ is the vocabulary size and $d_w$ is the word embedding dimension (e.g., 128, 256, 512)
  - More generally can embed any features
  - Example: $n$-grams, special indicators (language, beginning/end of a span, etc.)
- Efficient lookup operation

$E : \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \mapsto \begin{bmatrix} 0.7 & -0.1 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & -0.4 & 0.6 \end{bmatrix}$

- Very sparse updates: only a few columns of $E$ updated on a single batch
- Special padding token: $E(<\text{pad}>) = (0, \ldots, 0)$
Continuous Bag-of-Words (CBOW) Encoder

- \( \text{enc}_\theta : \mathcal{V}^+ \rightarrow \mathbb{R}^{d_w} \) defined by

\[
\text{enc}_\theta(x_1 \ldots x_T) = \frac{1}{T} \sum_{t=1}^{T} E(x_t)
\]

- Optionally apply additional layers (e.g.,
  \( \text{enc}_\theta(x_1 \ldots x_T) = \tanh(W \frac{1}{T} \sum_{t=1}^{T} E(x_t) + b)) \)

- Differentiable in \( E \) and can be fed into a linear classifier to learn end-to-end

- **Pros.** Simple, natural continuous extension of bag-of-words (BOW) representation (\( \approx \) feedforward on BOW)

- **Cons.** Like BOW, CBOW is incapable of modeling word ordering.
Convolutional Layer

- Idea: Slide an \( n\)-gram filter (aka. “kernel”) across text to identify a certain aspect from local patterns
- Example: Trigram filter \( F_3 \) “activates” at negative sentiment
  - \([\text{the movie was}] \) not super good \( \mapsto -0.2 \)
  - \([\text{movie was not}] \) super good \( \mapsto -0.1 \)
  - \([\text{movie was not super}] \) good \( \mapsto 0.3 \)
  - \([\text{movie was not super good}] \) \( \mapsto 1.8 \)

Take 1.8 is the final output of \( F_3 \) on the sentence (large value means the filter is activated).

- Sliding can be implemented efficiently in parallel.
- Learnable parameter: \( U \in \mathbb{R}^{3 \times d_w} \) defining

\[
F_3(x_1 \ldots x_T) = \max_{t=1}^{T-3+1} g \left( \sum_{i,j} \left[ U \right]_{3 \times d_w} \odot \left[ E(x_t, x_{t+1}, x_{t+2}) \right]_{3 \times d_w} \right)
\]

\( g \) is a nonlinear function (e.g., ReLU). \( U \) reused for all inputs.
Convolutional Neural Networks (CNNs)

- Learn $K$ “types” of trigram filter $F_3 = (F_3^{(1)} \ldots F_3^{(K)})$
- Each type expected to learn different aspects
  - Depends on the learning problem (e.g., positive and negative sentiments for sentiment classification)
  - Certain key phrases activate certain filters (“is good.”)
- Treated as a $K$-dimensional encoder $F_3 : \mathcal{V}^+ \rightarrow \mathbb{R}^K$.
- General CNNs: Multiple $n$-grams (e.g., $n \in \{3, 4, 5\}$) and concatenate outputs

$$\text{CNN}_{n \in \{3, 4, 5\}, K}(x_1 \ldots x_T) = \begin{bmatrix} F_3(x_1 \ldots x_T) \\ F_4(x_1 \ldots x_T) \\ F_5(x_1 \ldots x_T) \end{bmatrix} \in \mathbb{R}^{3K}$$

Treated as a $WK$-dimensional encoder where $W$ is the number of $n$ values
Variations of CNNs

- **Stacking**: multiple convolutional layers stacked on each other
- **Stride**: Skip ahead when sliding (previously stride 1), reduces output dim
- **Mean pooling**: Instead of taking max activation, average all activations?

Stride > 1 + stacking: learn progressively “higher-level” patterns

“Dilated” CNN with 3 conv layers, stride 2
Computer Vision and CNNs

- CNNs originated from image processing.

  ![Diagram of CNN architecture](image1)

- Motivation: translation invariance (can have bird appear anywhere in the image)

- Straightforward application to NLP by treating text as 1-dimensional image
  - Same motivation: can have “not good” appear anywhere

- **Cons.** Still cannot model word ordering beyond filter sizes.
Recurrent Neural Networks (RNNs)

- Idea: Read text $x_1 \ldots x_T$ (already word embeddings for notational convenience) left-to-right, updating internal state

\[
\begin{align*}
    h_0 &= (0, \ldots, 0) \\
    h_1 &= \text{RNN}_\theta(h_0, x_1) \\
    h_2 &= \text{RNN}_\theta(h_1, x_2) \\
    & \vdots \\
    h_T &= \text{RNN}_\theta(h_{T-1}, x_T)
\end{align*}
\]

- $\text{RNN}_\theta : \mathbb{R}^{d_h} \times \mathbb{R}^{d_w} \rightarrow \mathbb{R}^{d_h}$ is a feedforward ("RNN cell"), e.g.,

\[
\text{RNN}_\theta(h_{t-1}, x_t) = \tanh(Wx_t + Vh_{t-1} + b)
\]

- $h_t \in \mathbb{R}^{d_h}$: function of $x_1 \ldots x_t$

\[
h_t = \text{RNN}_\theta(\cdots \text{RNN}_\theta(\text{RNN}_\theta(0_d, x_1), x_2), \cdots , x_t)
\]
Stacked RNNs

- Number of RNN layers $K$
- Stacked RNN cell $\text{RNN}_\theta : \mathbb{R}^{Kd_h} \times \mathbb{R}^{d_w} \to \mathbb{R}^{Kd_h}$

\[ \text{RNN}_\theta(x_t, h_{t-1} = (h_{t-1}^{(1)} \ldots h_{t-1}^{(K)})) = (h_{t}^{(1)} \ldots h_{t}^{(K)}) \]

consisting of $K$ RNN cells

\[
\begin{align*}
  h_{t}^{(1)} &= \text{RNN}_{\theta}^{(1)}(h_{t-1}^{(1)}, x_t) & \text{RNN}_{\theta}^{(1)} : \mathbb{R}^{d_h} \times \mathbb{R}^{d_w} \to \mathbb{R}^{d_h} \\
  h_{t}^{(2)} &= \text{RNN}_{\theta}^{(2)}(h_{t-1}^{(2)}, h_{t}^{(1)}) & \text{RNN}_{\theta}^{(2)} : \mathbb{R}^{d_h} \times \mathbb{R}^{d_h} \to \mathbb{R}^{d_h} \\
  & \vdots \\
  h_{t}^{(K)} &= \text{RNN}_{\theta}^{(K)}(h_{t-1}^{(K)}, h_{t}^{(K-1)}) & \text{RNN}_{\theta}^{(K)} : \mathbb{R}^{d_h} \times \mathbb{R}^{d_h} \to \mathbb{R}^{d_h}
\end{align*}
\]

- Maps $x_1 \ldots x_T$ to $h_1 \ldots h_T \in \mathbb{R}^{Kd_h}$
  - Often just use the final layer $h_1^{(K)} \ldots h_T^{(K)} \in \mathbb{R}^{d_h}$
Exploding/Vanishing Gradient Problems

- In general, deep networks with multiplicative weights get gradients in exponential form

\[
\left| \frac{\partial w^n x}{\partial x} \right| = |w^n| \approx \begin{cases} 
0 & \text{if } |w| < 1 \text{ ("vanishes")}, \\
\infty & \text{if } |w| > 1 \text{ ("explodes")}
\end{cases}
\]

- RNN: if \( x_1 \ldots x_T \) is long, gradient of \( \text{Loss}(h_T) \) wrt. variables with small \( t \) will either vanish or explode.

- Solutions
  - Gradient clipping
  - Architectural modifications: maintain an extra "cell" state \( c_t \) that we carry over without losing signals (e.g., LSTM)
Long Short-Term Memory (LSTM) Cell

- Parameters (omitting bias terms) $U^q, U^c, U^o \in \mathbb{R}^{d_h \times d_w}$, $V^q, V^c, V^o, W^q, W^o \in \mathbb{R}^{d_h \times d_h}$

$$
q_t = \sigma (U^q x_t + V^q h_{t-1} + W^q c_{t-1}) \\
c_t = (1 - q_t) \odot c_{t-1} + q_t \odot \tanh (U^c x_t + V^c h_{t-1}) \\
o_t = \sigma (U^o x_t + V^o h_{t-1} + W^o c_t) \\
h_t = o_t \odot \tanh(c_t)
$$

- Idea: Memory cells $c_t$ can carry long-range information (image credit: Colah’s blog)

Network can choose to make $q_t = 0$!

- Cell states typically ignored: loss still defined using $h_t$
Bidirectional RNNs

Left-to-right RNN: $h_t$ is a function of $x_{t'}$ for $t' \leq t$ only

Bidirectional RNN: Make $h_t$ a function of all $x_1 \ldots x_T$ as follows.

- Left-to-right RNN:
  \[
  \overrightarrow{\text{RNN}}_\theta(x_1 \ldots x_T) = \overrightarrow{h}_1 \ldots \overrightarrow{h}_T
  \]

- Right-to-left RNN:
  \[
  \overleftarrow{\text{RNN}}_\theta(x_T \ldots x_1) = \overleftarrow{h}_T \ldots \overleftarrow{h}_1
  \]

- New representation of $x_t$
  \[
  h_t = (\overrightarrow{h}_t, \overleftarrow{h}_{T-t+1}) \in \mathbb{R}^{2d_h}
  \]

- Both RNNs learned jointly to optimize some loss (in $h_t$).

To get single vector, can average $h_1 \ldots h_T \in \mathbb{R}^{2d_h}$. 
Problems with Recurrent Architectures

Recurrent architectures are intuitive and effective.

- Compact recurrent cell updating an internal state is similar to how humans read text.
- Sensitive to word ordering
- Performs well, especially with LSTM cells and bidirectional variants

BUT

- **Slow.** Computation of $h_1 \ldots h_T$ cannot be parallelized
  - Must compute $h_{t-1}$ first before computing $h_t$.
- **Shallow bidirectionality.** Even if bidirectional, only a function of one side until the end of RNN computation
Attention

- Key recent progress in deep learning (originated from NLP)
- Idea: Let the model select which input vectors to use, by defining a distribution over them
- Three types of vector
  - **Query vector**: $q \in \mathbb{R}^d$
  - **Key/value vectors**: $(k_1, v_1), \ldots, (k_T, v_T) \in \mathbb{R}^d \times \mathbb{R}^d$
- Sometimes key/value collectively called **memory bank** $M$
- Compute an embedding from $M$ “attended” by $q$

\[
(p_1 \ldots p_T) = \text{softmax}(q^\top k_1, \ldots, q^\top k_T)
\]

\[
\text{Attn}(q, M) := \sum_{t=1}^{T} p_t v_t
\]
Attention in Matrix Form

Input

- $Q = (q_1 \ldots q_{T'}) \in \mathbb{R}^{T' \times d}$: $T'$ query vectors as rows
- $K = (k_1 \ldots k_T) \in \mathbb{R}^{T \times d}$: $T$ key vectors as rows
- $V = (v_1 \ldots v_T) \in \mathbb{R}^{T \times d}$: $T$ value vectors as rows

Output

- $A = (a_1 \ldots a_{T'}) \in \mathbb{R}^{T' \times d}$: $a_t = \text{Attn}(q_t, M = (K, V))$

Compute $A = \text{Attn}(Q, K, V)$ efficiently in matrix form:

$$A = \text{softmax}_{\text{row-wise}} (\underbrace{Q}_{T' \times d}, \underbrace{K^\top}_{d \times T}, \underbrace{V}_{T \times d})$$
Multi-Head Attention

- Idea: Make $H$ types of attention (“heads”)
  - May learn different attention behaviors.

  \[(0.7, 0.1, 0.1, 0.0)\] (head 1)
  \[(0.0, 0.1, 0.5, 0.4)\] (head 2)

- Unlike raw attention, there are learnable parameters.
  1. For each type $\tau \in \{q, k, v\}$: Linear function $f_{\theta}^{(\tau, i)} : \mathbb{R}^d \to \mathbb{R}^{d/H}$ for $i = 1 \ldots H$ (assume $d$ is divisible by $H$)
  2. Linear function $g_{\theta} : \mathbb{R}^d \to \mathbb{R}^d$

- Given $Q \in \mathbb{R}^{T' \times d}$ and $K, V \in \mathbb{R}^{T \times d}$, compute

$$
\text{Attn}^H_{\theta}(Q, K, V) = g_{\theta}\left( \bigoplus_{i=1}^{H} \text{Attn}^{(f_{\theta}^{(q,i)}(Q), f_{\theta}^{(k,i)}(K), f_{\theta}^{(v,i)}(V))}_{d/H} \right)
$$
Application: Self-Attention Encoder

- Initial word embeddings $X = (x_1 \ldots x_T) \in \mathbb{R}^{T \times d}$
- Want new embeddings $Z = (h_1 \ldots h_T) \in \mathbb{R}^{T \times d}$ such that $h_t$ is a function of all $x_1 \ldots x_T$
- **Self-attention**: Use $X$ as key, query, value at the same time!

$$Z = \text{Attn}^H_\theta(X, X, X)$$

- Multi-head attention parameters will “specialize” $X$ internally.
- Unlike recurrent, $h_t$ has direct connection to every input.
Transformer Encoder (Vaswani et al., 2017)

- Do self-attention many times, with other tricks like dropout, residual connection, layer normalization.
- There is no “right” implementation, other than the general importance of multiple applications of self-attention.
- Example: Given $X = Z_0 \in \mathbb{R}^{T \times d}$, for layer $l = 1 \ldots 6$, compute using layer-specific parameters

\[
N_{l-1} = \text{LayerNormalization}_\theta(Z_{l-1})
\]
\[
H_l = \text{Attn}_\theta^H(N_{l-1}, N_{l-1}, N_{l-1})
\]
\[
\hat{Z}_l = \text{Dropout}_{0.1}(H_l) + Z_{l-1}
\]
\[
Z_l = \text{NonlinearTransformation}(\hat{Z}_l)
\]

Use $Z_6 \in \mathbb{R}^{T \times d}$ as final $d$-dimensional embeddings of the input tokens.
- To get a single vector, can either average or take the first row.
Bidirectional RNNs vs Transformers

Transformer: “vertically recurrent”

- Intuition: Can refine its state for many rounds (multi-hop reasoning)

not bidirectional until later

deeply bidirectional
Self-Attention Visualization (Vaswani et al., 2017)

Layer 5 and 6, one of the “heads”

Different heads learn different weights
Details: Position Encodings, Masking

- Attention does not differentiate positions.

\[
\text{Attn}(q, ((k_1, k_2), (v_1, v_2))) = \text{Attn}(q, ((k_2, k_1), (v_2, v_1)))
\]

- Solution: Explicitly model positions at input level. Various approaches:
  - **Absolute positions**: Introduce an embedding \( \pi_i \in \mathbb{R}^d \) for positions \( i = 1, 2, \ldots \), and use \((x_1 + \pi_1, \ldots, x_T + \pi_T)\) as input.
  - **Relative positions**: Introduce an embedding \( \pi_i \in \mathbb{R}^d \) for offsets \( i = -k, \ldots, k \) between key and query and use it when computing logits between key-query pairs.

- If we don’t want \( q \) to attend to certain position \( i \), can specify that by “masking” logits

\[
\text{Mask}_{\{2,3\}}(q^\top k_1, q^\top k_2, q^\top k_3) = (q^\top k_1, -\infty, -\infty)
\]
Details: Scaled Dot Product, Training

- In practice we scale dot products by $1/\sqrt{d}$

\[
\text{Attn}(Q, K, V) = \text{softmax} \left( \frac{QK^\top}{\sqrt{d}} \right) V
\]

Helps stabilize the variance of dot products when $d$ is large.

- Training transformers can be nontrivial. Original work needed a very specific training setting
  - Xavier uniform initialization
  - Adam optimizer with non-default hyperparameters
  - Learning rate schedule: manually change learning rate during training (on top of Adam’s adaptive learning rate)
Summary

Input: word embeddings, learnable vectors for distinct word types

- **Averaging word embeddings**: Simplest way to embed text, but cannot model word ordering
- **CNNs**: Learn $n$-gram filters, still unable to model general word ordering
- **RNNs**: Recurrent updates naturally model word ordering, but cannot be parallelized and one-sided (or shallowly bidirectional)
- **Transformers**: Make all inputs attend to all inputs directly via self-attention, stacked to capture “deep” patterns (deeply bidirectional), can be parallelized and made sensitive to word ordering with position encodings, but tricky to train

Any of these encoders can be “plugged in” to optimize a task-specific loss!