Physical Sciences 2: Assignments for Oct. 22 - Oct 31, 2019:

Homework #7: Elasticity and Fluid Statics
Due Thursday, Oct 31st at 9:00AM

This assignment must be turned in by 9:00AM on Thursday, October 31st. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to your assigned section TA's box.

You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

After completing this homework, you should…

• Be able to describe what is meant by elasticity and how it relates to Hooke’s law
• Be able to explain why the Young’s modulus is necessary and why Hooke’s law cannot always be used
• Understand the stress-strain formula and be able to explain what all terms mean
• Be able to interpret a stress-strain plot
• Be able to explain the static properties of fluids: density, compressibility, and pressure
• Understand Pascal’s principle and be able to apply it
• Be able to explain how and why pressure in a fluid varies and the consequences (buoyancy)
• Know Archimedes’ Principle and be able to apply it to solve problems involving buoyant forces
Here are summaries of this module’s important concepts to help you complete this homework:

Module 7: Elasticity and Fluid Statics
Compiled by Kristina Callaghan

**Elasticity**
- any material can stretch due to its spring-like molecular bonds
  - elastic limit: maximum ΔL for which the material will return to initial length L; past this point, the object will permanently deform and/or break
  - the relationship between the force applied and the change in distance is
    \[ \frac{F}{A} = Y \cdot \frac{ΔL}{L} \]
- this has the same form as Hooke’s law:
  \[ F = \left( \frac{Y \cdot A}{L} \right) \cdot ΔL \]
  - positive RHS because F and ΔL in same direction
  - stress is the quantity \( \frac{F}{A} \)
  - strain is the quantity \( \frac{ΔL}{L} \)

**Fluid Statics**
- density \( \rho \) is the ratio of mass \( m \) and volume \( V \), and is an intrinsic property of the material
  \[ \rho = \frac{m}{V} \]
  Density of water: \( \rho_{\text{H2O}} = 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3 \)
- a fluid is incompressible, meaning its molecules are already as close to each other as possible without touching
- pressure \( P \) is the amount of force exerted over some cross-sectional area \( A \)
  \[ P = \frac{F}{A} \]
- Units: 1 Pascal (Pa) = 1 N/m²; 1 atm = 101.3 kPa; 1 atm = 760 mm Hg

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Section 8: Review

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**Elastic Region**
- strain is below the elastic limit
- force is applied
- cross-sectional area
- Young’s modulus: property of a material which is the same no matter the size or shape of object
- L: original length of material

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**Breaking Point**
- strain is above the elastic limit
- force applied
- cross-sectional area
- where force is applied
- \( k \): constant

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**Linear Region**
- strain is below the elastic limit
- force is applied
- cross-sectional area
- where force is applied
- \( k \): constant
Fluid Statics

- Pascal's Principle: pressure is transmitted undiminished in an enclosed static fluid.

Example: consider an object at depth h; above the liquid is some pressure $P_0$.

The pressure at depth $h$, denoted as $P_i$, is equal to the pressure $P_0$ above the liquid, plus the static fluid pressure $\rho g h$.

$$P_i = P_0 + \rho g h$$

This means that the pressure is the same at all points on a horizontal line in a fluid (all of these points are at the same depth).

- Archimedes Principle: a fluid exerts an upward buoyant force $F_B$ on an object in the fluid.

  - magnitude of $F_B$ equal to the weight of the fluid displaced by an object.

$$F_B = M_{\text{displaced}} \cdot g$$

$$F_B = (\rho_{\text{fluid}} \cdot V_{\text{displaced}}) g$$

$V_{\text{displaced}}$ is the volume that is actually under water.
0. Reflections on Last Assignment (1 pt)

Pick one question from Homework 4 that you found particularly difficult and

a) describe any mistakes or misunderstandings you made

b) describe the best strategies to ensure you learn from your mistakes and won’t have the same misunderstanding again

1. Let it Fall (1 pt) Explicitly show that an object of density $\rho$ will sink when submerged in a fluid of density $\rho_{\text{fluid}}$, where $\rho_{\text{fluid}} < \rho$, and find its acceleration. Assume the impact of the drag force is negligible.

2. Hanging by a Thread (2 pts). A spider spins a fine thread of spider silk between two posts a distance $L_0$ apart. The thread has a diameter of $d = 5 \times 10^{-6}$ m and an equilibrium length of $L_0$. When the spider (with mass $m = 2$ milligram) hangs from the center of the thread, the thread sags and stretches to support the weight of the spider, making an angle of $\theta = 10^\circ$ below the horizontal, as shown.

   a) Why must the thread sag when the spider hangs from it?

   b) Calculate the tensile strain $\varepsilon$ in the thread and the Young’s modulus $Y$ for the silk.

3. Under pressure (3 pts) Your blood pressure (typically reported as “120 over 80” or something like that) has two parts: the first number is the systolic pressure, which is the arterial pressure during the contraction of the heart, and the second number is the diastolic pressure, which is the arterial pressure when the heart is relaxed.

   a) Blood pressures are typically given in torr, or millimeters of mercury (1 torr = 1 mmHg). Using the fact that mercury has a density of 13.6 g/cm$^3$, derive a conversion factor between torr and pascals, and use it to convert 120 torr into Pa.

   b) These blood pressures are known as gauge pressures, meaning that they are the amount by which the blood pressure exceeds atmospheric pressure. The first measurement of blood pressure involved inserting a long cannula (needle-tipped tube) into the artery of a horse. The other end of the tube was left open to the atmosphere. If the horse’s systolic blood pressure were 150 mmHg, how high would the blood from that artery have risen in the cannula? Assume that blood has the same density as water.
c) As we’ll see when we study fluid dynamics, fluids flow from high pressure to low pressure. Using your results from parts a) and b), why must IV bags be placed at least 20 cm above a patient’s arm?

4. Head over heels (3 pts) Blood pressures are measured at the height of the heart. If the maximum (systolic) pressure at the brain drops below zero (gauge pressure), then no blood will reach the brain. This condition can lead to loss of consciousness. If the systolic pressure at the brain exceeds zero but the diastolic pressure does not, blood will reach the brain sporadically instead of continuously. This can cause dizziness, but usually not blackout.

a) What is the minimum pressure (measured at the heart) needed to avoid dizziness? Is this the minimum systolic or diastolic pressure?

b) A giraffe’s head is about 3 meters above its heart. The giraffe’s circulatory system must be specially adapted for its unusual anatomy. What must be the minimum diastolic pressure at the heart of a giraffe? If the systolic:diastolic ratio is the same for giraffes as for humans, what is the systolic pressure of a giraffe at the level of its heart?

c) Conversely, the blood pressure in your feet is greater than the pressure at your heart. Estimate the maximum (systolic) blood pressure in your feet, if the systolic pressure at your heart is 120 mmHg. This increased pressure can lead to swelling of the feet.

d) Estimate the maximum (systolic) blood pressure in giraffe’s feet, which are 2.5 meters below its heart. Giraffes have very tight elastic skin around their ankles to support this pressure and prevent swelling.