Recitation 17
Tuesday November 8, 2022

Directions  Go over problems from Recitation 16 if any were not covered before moving to this recitation.

1  Recap

1.1  Symmetric Matrices
Symmetric matrices are important to study, often appearing in applications such as the Hessian second derivative and optimization problems. A matrix $A \in \mathbb{R}^{n \times n}$ is symmetric if $A = A^\top$.

With real symmetric matrices, all eigenvalues $\lambda_i$ are real and eigenvectors can always be chosen to be orthogonal. We can then create the eigendecomposition

$$A = TDT^\top \quad \text{or equivalently} \quad A = \sum_{i=1}^n \lambda_i v_i v_i^\top$$

where $A$ is a real symmetric matrix, $T$ is the matrix of orthogonal eigenvectors, and $D$ is a diagonal matrix of eigenvalues.

1.2  Quadratic Functions as Matrices
Oftentimes, we want to express quadratic functions as matrices to solve in an optimization problem. Any quadratic function $f(x_1, ..., x_n)$ can be expressed as

$$x^\top A x + b^\top x$$

where $A$ is a symmetric matrix $\in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ are coefficient matrices, and $x = [x_1 ... x_n]^\top$ represents a vector of $n$ variables.
2 Exercises

Initially go over exercises from previous recitation not covered

1. True or False
   (a) If a matrix $A$ is symmetric and invertible, so is $A^{-1}$.
   (b) For any matrix $A$, the matrix $AA^T$ is symmetric.

2. Write the following equations in matrix-vector form $x^T A x + b^T x$
   (a) $4x^2 - 6xy + 2y^2 + 7x - 35y$
   (b) $\frac{5}{2}x^2 - 2xy - xz + 2y^2 + 3yz + \frac{5}{2}z^2 + 2x - 35y - 47z$

3. Solve the following linear ODE. Use Julia to calculate any inverses of matrices.
   \[
   \frac{dx(t)}{dt} = -6x(t) + 3g(t)
   \]
   \[
   \frac{dy(t)}{dt} = 4x(t) + 5y(t)
   \]