Boolean Functions

1. Summary
   - Dictator Testing
   - FKN Test

2. O^2

3. Problems

Def. r-query function tester
choose r queries a priori

Def. Local tester class \( \mathcal{L} \)
if \( f \in \mathcal{L} \), always accepts
rejects w/ probability \( \geq \lambda \cdot \text{dist}(f, \mathcal{L}) \)

Ex. BLR, 3 query test, linear functions.

Avoid anti-dictator, assume BLR+NAE
Com assume r-query, r-query, ..., r-query
into \( \text{max}(x, ..., x) \)-query

Lemma. \( f: \{-1,1\}^n \rightarrow \mathbb{R} \) is k-dictator, non-constant
\[ \Pr[f(x) = \lambda] > \frac{1}{2} \] \( \Rightarrow \) \( \lambda \geq \frac{1}{k} \)

FKN: \( f: \{-1,1\}^n \rightarrow \mathbb{R} \), \( W[f] \leq 15 \),
then \( f \) is \( O(\sqrt{n}) \)-close \( \Rightarrow \)

Prob. \( n = \infty \), \( \text{Var}[X]\) is small, other
\( W[f] \leq 1.6. \)

Problem 1 - manipulate rejection probability
Problem 2 - testing close to \( f \) is \( 15\) of \( f \)
Problem 3 - why no 2-query \( k \) of \( f \)

Answer 1 - Problem 1:

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Problem 2 - testing close to \( f \) is \( 15\) of \( f \)
Problem 3 - why no 2-query \( k \) of \( f \)

Answer 1 - Problem 1: