Describe Fermi surface by $k_{Fn}(\theta)$ (eg in $d=2$).

Index $n$ counts different sheets of Fermi surface.

1. Fermi velocity $\mathbf{v}_F(\theta)$
   
   Direction of $\mathbf{v}_F(\theta)$ is $\mathbf{FS}_n$ at $F$ point $\theta$ (much also specifies the orientation).

2. Landau parameter $F_{n'n}(\theta, \theta')$

3. Quasiparticle residue $Z_n(\theta)$

4. Possibly a Berry gauge connection
   
   Universal

All low energy properties of the Fermi liquid are determined by this data.

Non-Fermi liquid metals. Many examples of metals

- generically in $d=1$

- "Strange metal" normal state of HTc cuprates

\[ T \]

\[ C \] No $g\cdot\mathbf{p}$ in photoemission, linear $T$

\[ \rho(T) \text{ rises only to from very low } T \text{ (a few) } \]

\[ \text{high } T \text{ (} \approx 800 \text{ K), etc) } \]
near quantum critical points in metals, particularly in "heavy fermion" systems (e.g., CeCu₆AuₓYb₁₋ₓSi₂, CeRhIrSb₅). Intermediate-T regime of many correlated metals (some ruthenates, cobaltates...)

Matter of principle question: Can NFL ground states exist in d > 1?

We will show that the answer is yes by studying a very simple model.

As in the discussion of spin liquid models, we do not aim to construct a realistic model of any material to answer the matter of principle question.

Consider the same Kagome spin model studied as a system with a d=2 Z_2 spin liquid:

\[ \mathcal{H}_0 = -J \sum_{\langle r, r' \rangle} \sigma^x_{r} \sigma^x_{r'} - h \sum_{r, \alpha} \sigma^\alpha_{r} \]

In addition to these spins, introduce a separate set of electrons \( \epsilon_{\alpha} \) that sit at the centers of the Kagome triangles, i.e., on the honeycomb sites.
Assume the electrons are described by the Hamiltonian

\[ H = -t \sum_{ij} c_i^+ c_j + \mu \sum_{ij} c_i^+ c_i \]

The model clearly has a global $U(1)$ symmetry

\[ c_i \rightarrow e^{i\alpha} c_i \]

and a corresponding conserved electron number

\[ n = \sum_{i \in \text{site}} c_i^+ c_i \]

We will consider a generic filling per lattice site (not necessarily commensurate).

First consider small $t \ll J \hbar$.

At $t = 0$, we know the phase diagram of $H_0$.

\[ \text{FM} \rightarrow \mathbb{Z}_2 \rightarrow \text{QSL} \rightarrow h \mathbb{Z} \]

Now turn on a $t \neq 0$.

For small $h \mathbb{Z}$, the FM phase will continue to be stable, i.e., $\langle c_i^+ c_i \rangle \neq 0 = m$

Then to leading order

\[ H \rightarrow -mt \sum \frac{c_i^+ (c_j + h_i c_j)}{\mathcal{N}} - \mu \sum \frac{c_i^+ c_i}{\mathcal{N}} \]
Then to leading order

\[ H' \rightarrow -t \sum_{\langle ij \rangle} \left( c_i^T c_j - \frac{1}{2} \delta_{ij} \right) \]

\[ -\mu \sum_{\langle ij \rangle} c_i^T c_j \]

\[ \text{P} \text{ I} \text{ species will separately partially fill their bands and form Fermi surfaces.} \]

Thus we get an ordinary Fermi liquid metal consisting with the FM order.

What happens when \( \frac{\hbar}{J} \) is large?

To analyse we use the previous mapping of this model to decoupled Ising models.

\[ \text{Reminder: } \mathbf{B} = \prod_{p=0}^{\mathbf{P}} \mathbf{s}^2 \text{ commutes with } H_0 + H_c \]

and \[ [\mathbf{F}_p, \mathbf{F}_p'] = 0 \implies \text{can diagonalize } \mathbf{F}_p \]

simultaneously with \( H_0 + H_c \).

At least for small \( t \), we can argue that ground state is in sector with \( \Phi_p \neq \Phi_{p'} \) \( \forall \mathbf{p} \neq \mathbf{p'} \)

\( \implies \text{can solve by writing} \]

\[ \mathbf{F} = \mathbf{r}^T \mathbf{r} \]

and \[ \mathbf{r} \mathbf{r} = \mathbf{r} \]

where
Then $H^c_c = -J \sum \langle II' \rangle \frac{\alpha^2}{I} I + -h \sum c^I c_I$

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

Now make a change of variables and define

\[ d_I = c^I c_I \quad \text{for each } I. \]

Clearly $H^0 + H^c = -J \sum \langle II' \rangle \frac{\alpha^2}{I} I + -h \sum c^I c_I$

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

\[ \langle II' \rangle \]

Not that $\langle d_I, d_J \rangle = 0$, $\langle d_I, d^+_J \rangle = \delta_{IJ}$

so that the $d$-operators are ordinary fermion operators.

Further under the global $U(1)$, $d_I \rightarrow e^{i\theta} d_I$ so the conserved that $d_I$ carry fermion #.

As the $d_I$'s are decoupled from the $\Phi$-degree of freedom (in this sector where $g = 1$ $I$, $P$).
It is clear that at generic filling the d-fermions will form a Fermi surface.

As they carry global U(1) charge, this is a metallic state.

However, this is not a Fermi liquid in the conventional sense; if $\hbar/\delta J$ is large enough to be in the phase where $\langle \sigma^z \rangle = 0$.

To see this, consider the electron Green's function

$$\langle c_i^+ (b) c_j (b) \rangle = \langle d_i^+ (b) d_j^z (b) d_j^+ (0) d_j^z (0) \rangle \langle d_j^+ (0) d_j (0) \rangle^*$$

$$\langle d_i^+ (0) d_j^z (0) \rangle \langle d_j^+ (0) d_j (0) \rangle$$

same as in FL decays exponentially in space.

As $\gamma^z$ is disordered,

$$\langle \gamma_i^z (b) \gamma_j^z (b) \rangle$$

decays exponentially in space

& oscillates in time with a frequency $\Delta gap$ of the $\gamma^z$ excitation $\gamma_{\gamma^z}$

3) Can modify check that the electron spectral...
Function $A(E, \omega)$ has zero weight for $N < \Delta_{FZ}$.

i.e the system looks “gapped” in photo-emission or in tunneling.

But for transport/low-$T$ thermodynamics it behaves like an ordinary metal.

The q.p. residue $Z$ is strictly zero.

This is a simple (may be the simplest) example of a non-fermi liquid phase and is known as the Orthogonal Metal.