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INTRODUCTION

1.1 More is different

- The collective excitations of a many-body system can be viewed as particles. However, the properties of those particles can be very different from the properties of the particles that form the many-body system.
- Guessing is better than deriving.
- Limits of classical computing.
- Our vacuum is just a special material.

A quantitative change can lead to a qualitative change. This philosophy is demonstrated over and over again in systems that contain many particles (or many degrees of freedom), such as solids and liquids. The physical principles that govern a system of a few particles can be very different from the physical principles that govern the collective motion of many-body systems. New physical concepts (such as the concepts of fermions and gauge bosons) and new physical laws and principles (such as the law of electromagnetism) can arise from the correlations of many particles (see Chapter 10).

Condensed matter physics is a branch of physics which studies systems of many particles in the ‘condensed’ (i.e. solid or liquid) states. The starting-point of current condensed matter theory is the Schrödinger equation that governs the motion of a number of particles (such as electrons and nuclei). The Schrödinger equation is mathematically complete. In principle, we can obtain all of the properties of any many-body system by solving the corresponding Schrödinger equation.

However, in practice, the required computing power is immense. In the 1980s, a workstation with 32 Mbyte RAM could solve a system of eleven interacting electrons. After twenty years the computing power has increased by 100-fold, which allows us to solve a system with merely two more electrons. The computing power required to solve a typical system of 10^{23} interacting electrons is beyond the imagination of the human brain. A classical computer made by all of the atoms in our universe would not be powerful enough to handle the problem.⁵ Such an impossible computer could only solve the Schrödinger equation for merely about 100

⁵ It would not even have enough memory to store a single state vector of such a system.

particles.⁶ We see that an generic interacting many-body system is an extremely complex system. Practically, it is impossible to deduce all of its exact properties from the Schrödinger equation. So, even if the Schrödinger equation is the correct theory for condensed matter systems, it may not always be helpful for obtaining physical properties of an interacting many-body system.

Even if we do get the exact solution of a generic interacting many-body system, very often the result is so complicated that it is almost impossible to understand it in full detail. To appreciate the complexity of the result, let us consider a tiny interacting system of 200 electrons. The energy eigenvalues of the system are distributed in a range of about 200 eV. The system has at least 2^{200} energy levels. The level spacing is about $200 \text{ eV} / 2^{200} = 10^{-60} \text{ eV}$. Had we spent a time equal to the age of the universe in measuring the energy, then, due to the energy–time uncertainty relation, we could only achieve an energy resolution of order 10^{-33} eV . We see that the exact result of the interacting many-body system can be so complicated that it is impossible to check its validity experimentally in full detail.⁷ To really understand a system, we need to understand the connection and the relationship between different phenomena of a system. Very often, the Schrödinger equation does not directly provide such an understanding.

As we cannot generally directly use the Schrödinger equation to understand an interacting system, we have to start from the beginning when we are faced with a many-body system. We have to treat the many-body system as a black box, just as we treat our mysterious and unknown universe. We have to guess a low-energy effective theory that directly connects different experimental observations, instead of deducing it from the Schrödinger equation. We cannot assume that the theory that describes the low-energy excitations bears any resemblance to the theory that describes the underlying electrons and nuclei.

This line of thinking is very similar to that of high-energy physics. Indeed, the study of strongly-correlated many-body systems and the study of high-energy physics share deep-rooted similarities. In both cases, one tries to find theories that connect one observed experimental fact to another. (Actually, connecting one observed experimental fact to another is almost the definition of a physical theory.) One major difference is that in high-energy physics we only have one ‘material’ (our vacuum) to study, while in condensed matter physics there are many different materials which may contain new phenomena not present in our vacuum (such as fractional statistics, non-abelian statistics, and gauge theories with all kinds of gauge groups).

⁶ This raises a very interesting question—how does nature do its computation? How does nature figure out the state of 10^{23} particles one second later? It appears that the mathematics that we use is too inefficient. Nature does not do computations this way.

⁷ As we cannot check the validity of the result obtained from the Schrödinger equation in full detail, our belief that the Schrödinger equation determines all of the properties of a many-body system is just a *faith*.

1.2 'Elementary' particles and physics laws are emergent phenomena

- Emergence—the first principle of many-body systems.
- Origin of 'elementary' particles.
- Origin of the 'beauty' of physics laws. (Why nature behaves reasonably.)

Historically, in our quest to understand nature, we have been misled by a fundamental (and incorrect) assumption that the vacuum is empty. We have (incorrectly) assumed that matter placed in a vacuum can always be divided into smaller parts. We have been dividing matter into smaller and smaller parts, trying to discover the smallest 'elementary' particles—the fundamental building block of our universe. We have been believing that the physics laws that govern the 'elementary' particles must be simple. The rich phenomena in nature come from these simple physics laws.

However, many-body systems present a very different picture. At high energies (or high temperatures) and short distances, the properties of the many-body system are controlled by the interaction between the atoms/molecules that form the system. The interaction can be very complicated and specific. As we lower the temperature, depending on the form of the interaction between atoms, a crystal structure or a superfluid state is formed. In a crystal or a superfluid, the only low-energy excitations are collective motions of the atoms. Those excitations are the sound waves. In quantum theory, all of the waves correspond to particles, and the particle that corresponds to a sound wave is called a phonon.⁸ Therefore, at low temperatures, a new 'world' governed by a new kind of particle—phonons—emerges. The world of phonons is a simple and 'beautiful' world, which is very different from the original system of atoms/molecules.

Let us explain what we mean by 'the world of phonons is simple and beautiful'. For simplicity, we will concentrate on a superfluid. Although the interaction between atoms in a gas can be complicated and specific, the properties of emergent phonons at low energies are simple and universal. For example, all of the phonons have an energy-independent velocity, regardless of the form of the interactions between the atoms. The phonons pass through each other with little interaction despite the strong interactions between the atoms. In addition to the phonons, the superfluid also has another excitation called rotons. The rotons can interact with each other by exchanging phonons, which leads to a dipolar interaction with a force proportional to $1/r^4$. We see that not only are the phonons emergent, but even the physics laws which govern the low-energy world of the phonons and rotons are emergent. The emergent physics laws (such as the law of the dipolar interaction and the law of non-interacting phonons) are simple and beautiful.

⁸ A crystal has three kinds of phonons, while a superfluid has only one kind of phonon.

I regard the law of $1/r^4$ dipolar interaction to be beautiful because it is not $1/r^3$, or $1/r^{4.13}$, or one of billions of other choices. It is precisely $1/r^4$, and so it is fascinating to understand why it has to be $1/r^4$. Similarly, the $1/r^2$ Coulomb law is also beautiful and fascinating. We will explain the emergence of the law of dipolar interaction in superfluids in the first half of this book and the emergence of Coulomb's law in the second half of this book.

If our universe itself was a superfluid and the particles that form the superfluid were yet to be discovered, then we would only know about low-energy phonons. It would be very tempting to regard the phonon as an elementary particle and the $1/r^4$ dipolar interaction between the rotons as a fundamental law of nature. It is hard to imagine that those phonons and the law of the $1/r^4$ dipolar interaction come from the particles that are governed by a very different set of laws.

We see that in many-body systems the laws that govern the emergent low-energy collective excitations are simple, and those collective excitations behave like particles. If we want to draw a connection between a many-body system and our vacuum, then we should connect the low-energy collective excitations in the many-body system to the 'elementary' particles (such as the photon and the electron) in the vacuum. But, in the many-body system, the collective excitations are not elementary. When we examine them at short length scales, a complicated non-universal atomic/molecular system is revealed. Thus, in many-body systems we have collective excitations (also called quasiparticles) at low energies, and those collective excitations very often do not become the building blocks of the model at high energies and short distances. The theory at the atomic scale is usually complicated, specific, and unreasonable. The simplicity and the beauty of the physics laws that govern the collective excitations do not come from the simplicity of the atomic/molecular model, but from the fact that those laws have to allow the collective excitations to survive at low energies. A generic interaction between collective excitations may give those excitations a large energy gap, and those excitations will be unobservable at low energies. The interactions (or physics laws) that allow gapless (or almost gapless) collective excitations to exist must be very special—and 'beautiful'.

If we believe that our vacuum can be viewed as a special many-body material, then we have to conclude that there are no 'elementary' particles. All of the so-called 'elementary' particles in our vacuum are actually low-energy collective excitations and they may not be the building blocks of the fundamental theory. The fundamental theory and its building blocks at high energies⁹ and short distances are governed by a different set of physical laws. According to the point of view of emergence, those laws may be specific, non-universal, and complicated.

⁹ Here, by high energies we mean the energies of the order of the Planck scale $M_P = 1.2 \times 10^{19}$ GeV.

The beautiful world and reasonable physical laws at low energies and long distances emerge as a result of a ‘natural selection’: the physical laws that govern the low-energy excitations should allow those excitations to exist at low energies. In a sense, the ‘natural selection’ explains why our world is reasonable.

Someone who knows both condensed matter physics and high-energy physics may object to the above picture because our vacuum appears to be very different from the solids and liquids that we know of. For example, our vacuum contains Dirac fermions (such as electrons and quarks) and gauge bosons (such as light), while solids and liquids seemingly do not contain these excitations. It appears that light and electrons are fundamental and cannot be emergent. So, to apply the picture of emergence in many-body systems to elementary particles, we have to address the following question: can gauge bosons and Dirac fermions emerge from a many-body system? Or, more interestingly, can gauge bosons and Dirac fermions emerge from a many-boson system?

The fundamental issue here is where do fermions and gauge bosons come from? What is the origin of light and fermions? Can light and fermions be an emergent phenomenon? We know that massless (or gapless) particles are very rare in nature. If they exist, then they must exist for a reason. But what is the reason behind the existence of the massless photons and nearly massless fermions (such as electrons)? (The electron mass is smaller than the natural scale—the Planck mass—by a factor of 10^{22} and can be regarded as zero for our purpose.) Can many-body systems provide an answer to the above questions?

In the next few sections we will discuss some basic notions in many-body systems. In particular, we will discuss the notion that leads to gapless excitations and the notion that leads to emergent gauge bosons and fermions from local bosonic models. We will see that massless photons and massless fermions can be emergent phenomena.

1.3 Corner-stones of condensed matter physics

- Landau’s symmetry-breaking theory (plus the renormalization group theory) and Landau’s Fermi liquid theory form the foundation of traditional condensed matter physics.

The traditional many-body theory is based on two corner-stones, namely Landau’s Fermi liquid theory and Landau’s symmetry-breaking theory (Landau, 1937; Ginzburg and Landau, 1950). The Fermi liquid theory is a perturbation theory around a particular type of ground state—the states obtained by filling single-particle energy levels. It describes metals, semiconductors, magnets, superconductors, and superfluids. Landau’s symmetry-breaking theory points out that

the reason that different phases are different is because they have different symmetries. A phase transition is simply a transition that changes the symmetry. Landau's symmetry-breaking theory describes almost all of the known phases, such as solid phases, ferromagnetic and anti-ferromagnetic phases, superfluid phases, etc., and all of the phase transitions between them.

Instead of the origin of light and fermions, let us first consider a simpler problem of the origin of phonons. Using Landau's symmetry-breaking theory, we can understand the origin of the gapless phonon. In Landau's symmetry-breaking theory, a phase can have gapless excitations if the ground state of the system has a special property called spontaneous breaking of the continuous symmetry (Nambu, 1960; Goldstone, 1961). Gapless phonons exist in a solid because the solid breaks the continuous translation symmetries. There are precisely three kinds of gapless phonons because the solid breaks three translation symmetries in the x , y , and z directions. Thus, we can say that the origin of gapless phonons is the translational symmetry breaking in solids.

It is quite interesting to see that our understanding of a gapless excitation—phonon—is rooted in our understanding of the phases of matter. Knowing light to be a massless excitation, one may perhaps wonder if light, just like a phonon, is also a Nambu–Goldstone mode from a broken symmetry. However, experiments tell us that a gauge boson, such as light, is really different from a Nambu–Goldstone mode in $3 + 1$ dimensions.

In the late 1970s, we felt that we understood, at least in principle, all of the physics about phases and phase transitions. In Landau's symmetry-breaking theory, if we start with a purely bosonic model, then the only way to get gapless excitations is via spontaneous breaking of a continuous symmetry, which will lead to gapless *scalar bosonic* excitations. It seems that there is no way to obtain gapless gauge bosons and gapless fermions from symmetry breaking. This may be the reason why people think that our vacuum (with massless gauge bosons and nearly-gapless fermions) is very different from bosonic many-body systems (which were believed to contain only gapless scalar bosonic collective excitations, such as phonons). It seems that there does not exist any order that gives rise to massless light and massless fermions. Due to this, we put light and fermions into a different category to phonons. We regard them as elementary and introduce them by hand into our theory of nature.

However, if we really believe that light and fermions, just like phonons, exist for a reason, then such a reason must be a certain order in our vacuum that protects their masslessness.¹⁰ Now the question is what kind of order can give rise to light and fermions, and protect their masslessness? From this point of view, the very

¹⁰ Here we have already assumed that light and fermions are not something that we place in an empty vacuum. Our vacuum is more like an 'ocean' which is not empty. Light and fermions are collective excitations that correspond to certain patterns of 'water' motion.

existence of light and fermions indicates that our understanding of the states of matter is incomplete. We should deepen and expand our understanding of the states of matter. There should be new states of matter that contain new kinds of orders. The new orders will produce light and fermions, and protect their masslessness.

1.4 Topological order and quantum order

- There is a new world beyond Landau's theories. The new world is rich and exciting.

Our understanding of this new kind of order starts at an unexpected place—fractional quantum Hall (FQH) systems. The FQH states discovered in 1982 (Tsui *et al.*, 1982; Laughlin, 1983) opened a new chapter in condensed matter physics. What is really new in FQH states is that we have lost the two cornerstones of the traditional many-body theory. Landau's Fermi liquid theory does not apply to quantum Hall systems due to the strong interactions and correlations in those systems. What is more striking is that FQH systems contain many *different* phases at zero temperature which have the *same* symmetry. Thus, those phases cannot be distinguished by symmetries and cannot be described by Landau's symmetry-breaking theory. We suddenly find that we have nothing in the traditional many-body theory that can be used to tackle the new problems. Thus, theoretical progress in the field of strongly-correlated systems requires the introduction of new mathematical techniques and physical concepts, which go beyond the Fermi liquid theory and Landau's symmetry-breaking principle.

In the field of strongly-correlated systems, the developments in high-energy particle theory and in condensed matter theory really feed upon each other. We have seen a lot of field theory techniques, such as the nonlinear σ -model, gauge theory, bosonization, current algebra, etc., being introduced into the research of strongly-correlated systems and random systems. This results in a very rapid development of the field and new theories beyond the Fermi liquid theory and Landau's symmetry-breaking theory. This book is an attempt to cover some of these new developments in condensed matter theory.

One of the new developments is the introduction of quantum/topological order. As FQH states cannot be described by Landau's symmetry-breaking theory, it was proposed that FQH states contain a new kind of order—topological order (Wen, 1990, 1995). Topological order is new because it cannot be described by symmetry breaking, long-range correlation, or local order parameters. None of the usual tools that we used to characterize a phase apply to topological order. Despite this, topological order is not an empty concept because it can be characterized by a new set of tools, such as the number of degenerate ground states (Haldane

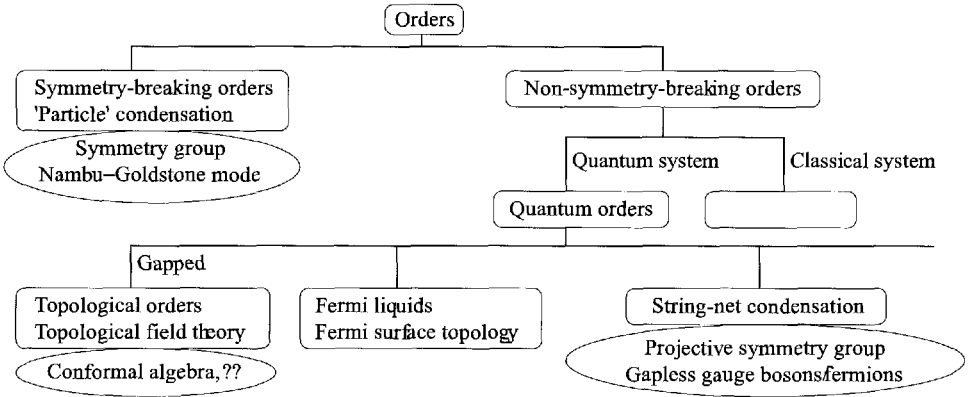


FIG. 1.1. A classification of different orders in matter (and in a vacuum).

and Rezayi, 1985), quasiparticle statistics (Arovas *et al.*, 1984), and edge states (Halperin, 1982; Wen, 1992).

It was shown that the ground-state degeneracy of a topologically-ordered state is robust against *any* perturbations (Wen and Niu, 1990). Thus, the ground-state degeneracy is a universal property that can be used to characterize a phase. The existence of topologically-degenerate ground states proves the existence of topological order. Topological degeneracy can also be used to perform fault-tolerant quantum computations (Kitaev, 2003).

The concept of topological order was recently generalized to quantum order (Wen, 2002c) to describe new kinds of orders in gapless quantum states. One way to understand quantum order is to see how it fits into a general classification scheme of orders (see Fig. 1.1). First, different orders can be divided into two classes: symmetry-breaking orders and non-symmetry-breaking orders. The symmetry-breaking orders can be described by a local order parameter and can be said to contain a condensation of point-like objects. The amplitude of the condensation corresponds to the order parameter. All of the symmetry-breaking orders can be understood in terms of Landau's symmetry-breaking theory. The non-symmetry-breaking orders cannot be described by symmetry breaking, nor by the related local order parameters and long-range correlations. Thus, they are a new kind of order. If a quantum system (a state at zero temperature) contains a non-symmetry-breaking order, then the system is said to contain a non-trivial quantum order. We see that a quantum order is simply a non-symmetry-breaking order in a quantum system.

Quantum orders can be further divided into many subclasses. If a quantum state is gapped, then the corresponding quantum order will be called the topological order. The low-energy effective theory of a topologically-ordered state will

be a topological field theory (Witten, 1989). The second class of quantum orders appears in Fermi liquids (or free fermion systems). The different quantum orders in Fermi liquids are classified by the Fermi surface topology (Lifshitz, 1960). The third class of quantum orders arises from a condensation of nets of strings (or simply string-net condensation) (Wen, 2003a; Levin and Wen, 2003; Wen, 2003b). This class of quantum orders shares some similarities with the symmetry-breaking orders of ‘particle’ condensation.

We know that different symmetry-breaking orders can be classified by symmetry groups. Using group theory, we can classify all of the 230 crystal orders in three dimensions. The symmetry also produces and protects gapless collective excitations—the Nambu–Goldstone bosons—above the symmetry-breaking ground state. Similarly, different string-net condensations (and the corresponding quantum orders) can be classified by mathematical object called projective symmetry group (PSG) (Wen, 2002c). Using PSG, we can classify over 100 different two-dimensional spin liquids that all have the same symmetry. Just like the symmetry group, the PSG can also produce and protect gapless excitations. However, unlike the symmetry group, the PSG produces and protects gapless gauge bosons and fermions (Wen, 2002a,c; Wen and Zee, 2002). Because of this, we can say that light and massless fermions can have a unified origin; they can emerge from string-net condensations.

In light of the classification of the orders in Fig. 1.1, this book can be divided into two parts. The first part (Chapters 3–5) deals with the symmetry-breaking orders from ‘particle’ condensations. We develop the effective theory and study the physical properties of the gapless Nambu–Goldstone modes from the fluctuations of the order parameters. This part describes ‘the origin of sound’ and other Nambu–Goldstone modes. It also describes the origin of the law of the $1/r^4$ dipolar interaction between rotons in a superfluid. The second part (Chapters 7–10) deals with the quantum/topological orders and string-net condensations. Again, we develop the effective theory and study the physical properties of low-energy collective modes. However, in this case, the collective modes come from the fluctuations of condensed string-nets and give rise to gauge bosons and fermions. So, the second part provides ‘an origin of light and electrons’, as well as other gauge bosons and fermions. It also provides an origin of the $1/r^2$ Coulomb law (or, more generally, the law of electromagnetism).

1.5 Origin of light and fermions

- The string-net condensation provides an answer to the origin of light and fermions. It unifies gauge interactions and Fermi statistics.

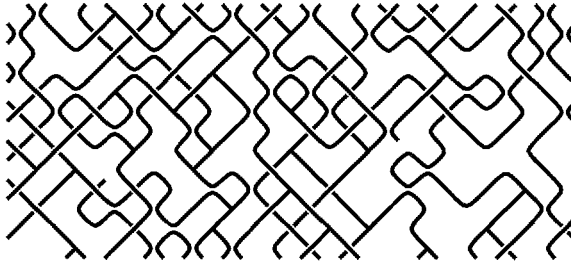


FIG. 1.2. Our vacuum may be a state filled with a string-net. The fluctuations of the string-net give rise to gauge bosons. The ends of the strings correspond to electrons, quarks, etc.

We used to believe that, to have light and fermions in our theory, we have to introduce by hand a fundamental $U(1)$ gauge field and anti-commuting fermion fields, because at that time we did not know of any collective modes that behave like gauge bosons and fermions. However, due to the advances over the last twenty years, we now know how to construct *local bosonic systems* that have emergent *unconfined* gauge bosons and/or fermions (Foerster *et al.*, 1980; Kalmeyer and Laughlin, 1987; Wen *et al.*, 1989; Read and Sachdev, 1991; Wen, 1991a; Moessner and Sondhi, 2001; Motrunich and Senthil, 2002; Wen, 2002a; Kitaev, 2003; Levin and Wen, 2003). In particular, one can construct ugly bosonic spin models on a cubic lattice whose low-energy effective theory is the beautiful quantum electrodynamics (QED) and quantum chromodynamics (QCD) with emergent photons, electrons, quarks, and gluons (Wen, 2003b).

This raises the following issue: do light and fermions in nature come from a fundamental $U(1)$ gauge field and anti-commuting fields as in the $U(1) \times SU(2) \times SU(3)$ standard model, or do they come from a particular quantum order in our vacuum? Is Coulomb's law a fundamental law of nature or just an emergent phenomenon? Clearly, it is more natural to assume that light and fermions, as well as Coulomb's law, come from a quantum order in our vacuum. From the connections between string-net condensation, quantum order, and massless gauge/fermion excitations, we see that string-net condensation provides a way to unify light and fermions. It is very tempting to propose the following possible answers to the three fundamental questions about light and fermions.

What are light and fermions?

Light is the fluctuation of condensed string-nets (of arbitrary sizes). Fermions are ends of condensed strings.

Where do light and fermions come from?

Light and fermions come from the collective motions of string-nets that fill the space (see Fig. 1.2).

Why do light and fermions exist?

Light and fermions exist because our vacuum happens to have a property called string-net condensation.

Had our vacuum chosen to have ‘particle’ condensation, then there would be only Nambu–Goldstone bosons at low energies. Such a universe would be very boring. String-net condensation and the resulting light and fermions provide a much more interesting universe, at least interesting enough to support intelligent life to study the origin of light and fermions.

1.6 Novelty is more important than correctness

- The Dao that can be stated cannot be eternal Dao. The Name that can be named cannot be eternal Name. The Nameless is the origin of universe. The Named is the mother of all matter.¹¹
- What can be stated cannot be novel. What cannot be stated cannot be correct.

In this introduction (and in some parts of this book), I hope to give the reader a sense of where we come from, where we stand, and where we are heading in theoretical condensed matter physics. I am not trying to summarize the generally accepted opinions here. Instead, I am trying to express my personal and purposely exaggerated opinions on many fundamental issues in condensed matter physics and high-energy physics. These opinions and pictures may not be correct, but I hope they are stimulating. From our experience of the history of physics, we can safely assume that none of the current physical theories are completely correct. (According to Lao Zi, the theory that can be written down cannot be the eternal theory, because it is limited by the mathematical symbols that we used to write down the theory.) The problem is to determine in which way the current theories are wrong and how to fix them. Here we need a lot of imagination and stimulation.

¹¹ These are the first four sentences of *Dao de jing* written by a Chinese philosopher Lao Zi over 2500 years ago. The above is a loose direct translation. Dao has meanings of ‘way’, ‘law’, ‘conduct’, etc. There are many very different translations of *Dao de jing*. It is interesting to search the Web and compare those different translations. The following is a translation in the context of this book. ‘The physical theory that can be formulated cannot be the final ultimate theory. The classification that can be implemented cannot classify everything. The unformulatable ultimate theory does exist and governs the creation of the universe. The formulated theories describe the matter we see every day.’

1.7 Remarks: evolution of the concept of elementary particles

- As time goes by, the status of elementary particles is downgraded from the building blocks of everything to merely collective modes of, possibly, a lowly bosonic model.

The Earth used to be regarded as the center of the universe. As times went by, its status was reduced to merely one of the billions of planets in the universe. It appears that the concept of elementary particles may have a similar fate.

At the beginning of human civilization, people realized that things can be divided into smaller and smaller parts. Chinese philosophers theorized that the division could be continued indefinitely, and hence that there were no elementary particles. Greek philosophers assumed that the division could not be continued indefinitely. As a result, there exist ultimate and indivisible particles—the building blocks of all matter. This may be the first concept of elementary particles. Those ultimate particles were called *atomos*. A significant amount of scientific research has been devoted to finding these *atomos*.

Around 1900, chemists discovered that all matter is formed from a few dozen different kinds of particles. People jumped the gun and named them atoms. After the discovery of the electron, people realized that elementary particles are smaller than atoms. Now, many people believe that photons, electrons, quarks, and a few other particles are elementary particles. Those particles are described by a field theory which is called the $U(1) \times SU(2) \times SU(3)$ standard model.

Although the $U(1) \times SU(2) \times SU(3)$ standard model is a very successful theory, now most high-energy physicists believe that it is not the ultimate theory of everything. The $U(1) \times SU(2) \times SU(3)$ standard model may be an effective theory that emerges from a deeper structure. The question is from which structure may the standard model emerge?

One proposal is the grand unified theories in which the $U(1) \times SU(2) \times SU(3)$ gauge group is promoted to $SU(5)$ or even bigger gauge groups (Georgi and Glashow, 1974). The grand unified theories group the particles in the $U(1) \times SU(2) \times SU(3)$ standard model into very nice and much simpler structures. However, I would like to remark that I do not regard the photon, electron, and other elementary particles to be emergent within the grand unified theories. In the grand unified theories, the gauge structure and the Fermi statistics were fundamental in the sense that the only way to have gauge bosons and fermions was to introduce vector gauge fields and anti-commuting fermion fields. Thus, to have the photon, electron, and other elementary particles, we had to introduce by hand the corresponding gauge fields and fermion fields. Therefore, the gauge bosons and fermions were added by hand into the grand unified theories; they did not emerge from a simpler structure.

The second proposal is the superstring theory (Green *et al.*, 1988; Polchinski, 1998). Certain superstring models can lead to the effective $U(1) \times SU(2) \times SU(3)$ standard model plus many additional (nearly) massless excitations. The gauge bosons and the graviton are emergent because the superstring theory itself contains no gauge fields. However, the Fermi statistics are not emergent. The electron and quarks come from the anti-commuting fermion fields on a $(1+1)$ -dimensional world sheet. We see that, in the superstring theory, the gauge bosons and the gauge structures are not fundamental, but the Fermi statistics and the fermions are still fundamental.

Recently, people realized that there might be a third possibility—string-net condensation. Banks *et al.* (1977) and Foerster *et al.* (1980) first pointed out that light can emerge as low-energy collective modes of a local bosonic model. Levin and Wen (2003) pointed out

that even three-dimensional fermions can emerge from a local bosonic model as the ends of condensed strings. Combining the two results, we find that the photon, electron, quark, and gluon (or, more precisely, the QED and the QCD part of the $U(1) \times SU(2) \times SU(3)$ standard model) can emerge from a local bosonic model (Wen, 2002a, 2003b) if the bosonic model has a string-net condensation. This proposal is attractive because the gauge bosons and fermions have a unified origin. In the string-net condensation picture, neither the gauge structure nor the Fermi statistics are fundamental; all of the elementary particles are emergent.

However, the third proposal also has a problem: we do not yet know how to produce the $SU(2)$ part of the standard model due to the chiral fermion problem. There are five deep mysteries in nature, namely, identical particles, Fermi statistics, gauge structure, chiral fermions, and gravity. The string-net condensation only provides an answer to the first three mysteries; there are two more to go.