Let $E_1, E_2$ events, $R: S \rightarrow R$,
$R(x) = \#(y \in S \text{ such that } x \in y)$

$E[R] = \sum_{E_1 \in E} R[E_1]$ is an $R$.

Expected Value of Random Variables (let independent random variables)

If $R, R'$ are independent for all $r, r'$, $\text{Pr}(R=r) P[R'=r']$.

$E[R,R'] = E[R] E[R']$.

\[
E[R] = \sum_r r \text{ Pr}(R=r)
\]

\[
E[R,R'] = \sum_{r,r'} r' \text{ Pr}(R=r, R'=r')
\]

\[
= \sum_r \sum_{r'} r' \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= \sum_r \text{ Pr}(R=r) \sum_{r'} r' \text{ Pr}(R=r')
\]

\[
= \text{ Pr}(R=r) E[R=r']
\]

\[
= E[R]
\]

\[
E[R,R'] = \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
E[R,R'] = \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= E[R] E[R']
\]

\[
E[R,R'] = \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= E[R] E[R']
\]

\[
E[R,R'] = \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= E[R] E[R']
\]

\[
E[R,R'] = \text{ Pr}(R=r) \text{ Pr}(R=r')
\]

\[
= E[R] E[R']
\]