Observe also:

- Dissipated power/area = \( \Delta \omega \)
  
  \[ = \pi \sum (\Delta L)^2 \]

So, in annulus \( 2\pi r \Delta r \) \( \Rightarrow \)

- Dissipated Energy = \( 2\pi r \Delta \omega (\Delta L)^2 \)
  
  \[ = \frac{3}{2} \frac{GM\dot{M}}{\sqrt{2}} \Delta r \quad \text{as above} \]

\[ \left[ \text{recall} \quad 3\pi r \sum = \dot{M} \left( 1 - \frac{c^2}{c^2} \right)^{\frac{3}{2}} \right] \]

and

\[ \text{Larmor \& Grav Energy Released} = \frac{GM\dot{M}}{2 \sqrt{2}} \Delta r \]

So:

- \( \Delta \omega \) \( \Rightarrow \) Grav energy released
- \( \frac{3}{2} \) \( \Rightarrow \) dissipated energy
The magnetic field and its properties recall:

\[ B = \frac{\mu_0 I}{2\pi r} \]

\[ \frac{dB}{dr} = \frac{\mu_0 I}{4\pi r^2} \]

- **Temperature**

\[ D(r) \text{ power/area} = \gamma \sum R^2 (r')^2 \]

so for black body:

\[ D(r) = 2 \sigma T^4 \]

\[ r \to \text{surface temperature} \]

\[ T_b^4 = \frac{3c}{8\pi G \mu_0} \left( \frac{M}{r^2} \right)^{1/2} \left( 1 - \left( \frac{r_s}{r} \right)^{3/2} \right) \]
\[ T_{\text{disk}} = \left( \frac{3 \, GM^2}{8t} \frac{\bar{v}}{\sqrt{3} \bar{V}} \right)^{1/4} \]

- no explicit \( r \) dependence!
- implicit via \( \bar{V} \).

See Aingle, Lodato for Spectral Energy Distributions.

→ Time Scales — Aperture

It's convenient now to review time scales and their ordering.

Recall — used velocity scales to construct time scale ordering
- simplified transport equations to obtain \( V_r \), mass diff.
Time Scales:

1. dynamical, rotation
2. vertical
3. thermal, cooling
4. viscous, functions of radius

\[ \Omega \sim \Omega_1 \sim \left( \frac{r^3}{GM} \right)^{1/2} \]

\[ \Omega \sim \Omega_1 \sim \Omega_2 \sim \left( \frac{r^3}{GM} \right)^{1/2} \]

Note: \( \Omega_{rot} \sim \Omega_{vert} \)

6. Cooling

[Thermal Energy Density]

Cooling \( \sim \) Thermal Energy Density / viscous heating rate

\( \sim \) necessary at equilibrium.
Thermodynamic Energy Density \( \sim \frac{P}{\Delta - 1} \)

\( \frac{P}{\rho} \sim \left( \frac{C_v}{\rho} \right)^\Delta \)

Then:

\[
\begin{align*}
\sum T E O \sim & \Omega \sum C_s^2 / \delta (\delta - 1) \\
C_s^2 = & \frac{\partial P}{\partial \delta}
\end{align*}
\]

Heating Rate:

\[
\nu \sum (\Omega \delta')^2
\]

\[\text{cooking} \sim \frac{\Omega \sum C_s^2 / \delta (\delta - 1)}{\nu \sum (\Omega \delta')^2}\]

\[\nu \delta \Omega = x \Sigma H^2 \]

\[\nu \delta \Sigma H = x \Omega H\]

\[\text{extensive discussion coming}\]

\[\text{cooking} \sim \frac{4}{\pi^2 (\Omega \delta)} \times \Omega\]
\[ \tau_{\text{visc}} = \frac{3}{2} \frac{r^3}{v}. \]

and have ordering:

\[ \tau_{\text{dyn}} < \tau_{\text{vert}} < \tau_{\text{th}} < \tau_{\text{visc}} \]

\text{N.B.: } Disk acquires slowly, maintains thermal equilibrium, with quasi-static structure.

\text{N.B.: } All time scales dependent

- Outer disk evolves more slowly.
Disk Outbursts and Transport Instabilities.

We have extensively discussed disk structure.

\[ T_\phi = \sum n r \frac{\partial T}{\partial q} \]

via opacity.

- Have said \( T_\phi \) about (structure and dependencies).

- Is this house-of-cards internally consistent?

- \( \Sigma \) dependence.

Now recall:

\[ \frac{d \Sigma}{dt} = \frac{3}{2} n \frac{\partial}{\partial r} \left[ r^{-1/2} \Sigma (r^{-3/2} \Sigma) \right] \]
\[ \dot{M} = -2\pi r v_r \Sigma \]

\[ \Sigma \rightarrow M = \Sigma \int \]

\[ \frac{\partial M}{\partial t} = \frac{\partial}{\partial \Sigma} \left( \Sigma \frac{\partial M}{\partial \Sigma} \right) \]

- diffusion edge

\[ \frac{\partial M}{\partial t} = \int \frac{\partial}{\partial \Sigma} \left( \Sigma \frac{\partial M}{\partial \Sigma} \right) \]

\[ \int \]

\[ \text{positive diffusion?} \]

\[ \text{negative diffusion?} \]

\[ \Rightarrow \text{instability} \]

\[ \frac{\partial \Sigma}{\partial \Sigma} \text{ is critical!} \]
Modelling (e.g. Bell, Lin '94) suggests

\[ \sum \]

\[ \rightarrow \]

\[ S \text{-} \text{curved} \]

\[ \rightarrow \text{bifurcation and back-transition} \]

\[ \rightarrow \text{LCO} \leftrightarrow \text{Disk outbursts/cycles} \]

TBC.