51. Patricia pays \$1.19 each to download songs to her digital media player. If \( n \) is the number of downloaded songs, which equation represents the cost \( C \) in dollars?

A \( C = 1.19n \)  
B \( n = 1.19C \)  
C \( C = 1.19 \times n \)  
D \( C = n + 1.19 \)

52. Suppose that \( y \) varies directly as \( x \), and \( y = 8 \) when \( x = 6 \). What is the value of \( y \) when \( x = 8 \)?

F 6  
G 12  
H \( \frac{10}{3} \)  
J 16

53. What is the relationship between the input \((x)\) and output \((y)\)?

A The output is two more than the input.  
B The output is two less than the input.  
C The output is twice the input.  
D The output is half the input.

54. SHORT RESPONSE A telephone company charges \$40 per month plus \$0.07 per minute. How much would a month of service cost a customer if the customer talked for 200 minutes?

Spiral Review

55. TELEVISION The graph shows the average number of television channels American households receive. What was the annual rate of change from 2004 to 2008? Explain the meaning of the rate of change. (Lesson 3-5)

Solve each equation. (Lesson 3-2)

56. \( 0 = 18 - 9x \)  
57. \( 2x + 14 = 0 \)  
58. \(-4x + 16 = 0 \)  
59. \(-5x - 20 = 0 \)  
60. \( 8x - 24 = 0 \)  
61. \( 12x - 144 = 0 \)

Evaluate each expression if \( a = 4, b = -2, \) and \( c = -4 \). (Lesson 3-5)

62. \( [2a + c] + 1 \)  
63. \( 4a - [3b + 2] \)  
64. \(-[a + 1] + 3c \)  
65. \( -a + [2 - a] \)  
66. \( |c - 2b| - 3 \)  
67. \(-2[3b - 8] \)

Skills Review

Find each difference.

68. \( 13 - (-1) \)  
69. \( 4 - 16 \)  
70. \(-3 - 3 \)  
71. \(-8 - (-2) \)  
72. \( 16 - (-10) \)  
73. \(-8 - 4 \)
**Example 1** Identify Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. Explain.

a. \(-4, -2, 0, 2, \ldots\)

\[
\begin{array}{cccc}
-4 & -2 & 0 & 2 \\
+2 & +2 & & \\
\end{array}
\]

The difference between terms in the sequence is constant. Therefore, this sequence is arithmetic.

b. \(1, 5, 9, 13, \ldots\)

\[
\begin{array}{cccc}
1 & 5 & 9 & 13 \\
+4 & +4 & +4 & \\
\end{array}
\]

This is not an arithmetic sequence. The difference between terms is not constant.

**GuidedPractice**

1A. \(-26, -22, -18, -14, \ldots\)

1B. \(1, 4, 9, 25, \ldots\)

You can use the common difference of an arithmetic sequence to find the next term.

**Example 2** Find the Next Term

Find the next three terms of the arithmetic sequence 15, 9, 3, \(-3, \ldots\)

**Step 1** Find the common difference by subtracting successive terms.

\[
\begin{array}{cccc}
15 & 9 & 3 & -3 \\
-6 & -6 & -6 & -6 \\
\end{array}
\]

The common difference is \(-6\).

**Step 2** Add \(-6\) to the last term of the sequence to get the next term.

\[
\begin{array}{cccc}
15 & 9 & 3 & -3 \\
-6 & -6 & -6 & -6 \\
\end{array}
\]

The next three terms in the sequence are \(-9, -15, \text{ and } -21\).

**GuidedPractice**

2. Find the next four terms of the arithmetic sequence 9, 5, 11, 0, 12.5, 14.0, \ldots\)

Each term in an arithmetic sequence can be expressed in terms of the first term \(a_1\) and the common difference \(d\).

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>In Terms of (a_1) and (d)</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>first term</td>
<td>(a_1)</td>
<td>(a_1)</td>
<td>8</td>
</tr>
<tr>
<td>second term</td>
<td>(a_2)</td>
<td>(a_1 + d)</td>
<td>8 + 1(3) = 11</td>
</tr>
<tr>
<td>third term</td>
<td>(a_3)</td>
<td>(a_1 + 2d)</td>
<td>8 + 2(3) = 14</td>
</tr>
<tr>
<td>fourth term</td>
<td>(a_4)</td>
<td>(a_1 + 3d)</td>
<td>8 + 3(3) = 17</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>nth term</td>
<td>(a_n)</td>
<td>(a_1 + (n-1)d)</td>
<td>8 + (n - 1)(3)</td>
</tr>
</tbody>
</table>

**KeyConcept** \(n\)th Term of an Arithmetic Sequence

The \(n\)th term of an arithmetic sequence with first term \(a_1\) and common difference \(d\) is given by \(a_n = a_1 + (n-1)d\), where \(n\) is a positive integer.

---

**Example 3** Find the \(n\)th Term

a. Write an equation for the \(n\)th term of the arithmetic sequence \(-12, -8, -4, 0, \ldots\).

**Step 1** Find the common difference.

\[
\begin{array}{cccc}
-12 & -8 & -4 & 0 \\
-4 & -4 & -4 & -4 \\
\end{array}
\]

The common difference is \(4\).

**Step 2** Write an equation.

\[
a_n = a_1 + (n - 1)d
\]

For the \(n\)th term

\[
a_n = -12 + (n - 1)4
\]

Distributive Property

\[
a_n = -12 + 4n - 4
\]

Simplify

\[
a_n = 4n - 16
\]

b. Find the 9th term of the sequence.

Substitute 9 for \(n\) in the formula for the \(n\)th term.

\[
a_9 = 4(9) - 16
\]

Multiply

\[
a_9 = 36 - 16
\]

Simplify

\[
a_9 = 20
\]

c. Graph the first five terms of the sequence.

<table>
<thead>
<tr>
<th>(n)</th>
<th>(4n - 16)</th>
<th>(a_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12</td>
<td>((-12))</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
<td>((-8))</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>((-4))</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>((0))</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>((4))</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
-12 & -8 & -4 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
-16 & -12 & -8 & 0 \\
\end{array}
\]

d. Which term of the sequence is 32?

In the formula for the \(n\)th term, substitute 32 for \(a_n\).

\[
a_n = 4n - 16
\]

For the \(n\)th term

\[
a_9 = 32
\]

Add 16 to each side

\[
48 = 4n
\]

Divide each side by 4

\[
n = 12
\]

**GuidedPractice**

Consider the arithmetic sequence \(-5, -10, -15, \ldots\)

3A. Write an equation for the \(n\)th term of the sequence.

3B. Find the 15th term in the sequence.

3C. Graph the first five terms of the sequence.

3D. Which term of the sequence is \(-115\)?
**Arithmetic Sequences and Functions**

As you can see from Example 3, the graph of the first five terms of the arithmetic sequence lie on a line. An arithmetic sequence is a linear function in which the independent variable, \( n \), is the domain and \( a_n \) is the range of the function. The common difference, \( d \), is the slope of the line. The formula can be rewritten as the function \( f(n) = a_1 + (n-1)d \), where \( n \) is the counting number.

While the domain of most linear functions are all real numbers, in Example 3 the domain of the function is the set of counting numbers and the range of the function is the set of integers on the line.

**Real-World Example 4: Arithmetic Sequences as Functions**

**INVITATIONS** Mariel is mailing invitations to her quinceañera. The arithmetic sequence \$0.42, \$0.84, \$1.26, \$1.68, \ldots \) represents the cost of postage.

- **a.** Write a function to represent this sequence.
  
  The first term, \( a_1 \), is 0.42. Find the common difference.
  
  \[
  0.42 + 0.42 = 0.84
  \]
  
  The common difference is 0.42.
  
  \[
  a_n = a_1 + (n-1)d
  \]
  
  Formula for the \( n \)th term
  
  \[
  = 0.42 + (n-1)0.42
  \]
  
  Distributive Property
  
  \[
  = 0.42 + 0.42n - 0.42
  \]
  
  The function is \( f(n) = 0.42n \).

- **b.** Graph the function and determine the domain.
  
  The rate of change of the function is 0.42. Make a table and plot points.
  
<table>
<thead>
<tr>
<th>( n )</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.42</td>
</tr>
<tr>
<td>2</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>1.26</td>
</tr>
<tr>
<td>4</td>
<td>1.68</td>
</tr>
<tr>
<td>5</td>
<td>2.10</td>
</tr>
</tbody>
</table>

  The domain of a function is the number of invitations Mariel mails. So, the domain is \( \{0, 1, 2, 3, \ldots\} \).

**Guided Practice**

4. **TRACK** The chart below shows the length of Martin's long jumps.

<table>
<thead>
<tr>
<th>Jump</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>8</td>
<td>9.5</td>
<td>11</td>
<td>12.5</td>
</tr>
</tbody>
</table>

- **A.** Write a function to represent this arithmetic sequence.
- **B.** Then graph the function.

---

**Check Your Understanding**

- **Example 1** Determine whether each sequence is an arithmetic sequence. Write yes or no.
  
  1. 15, 16, 15, 13, \ldots
  2. 4, 9, 14, 19, \ldots
  
  Explain.

- **Example 2** Find the next three terms of each arithmetic sequence.
  
  1. 12, 9, 6, 3, \ldots
  2. 1.2, 0.6, 0, -0.4, \ldots

- **Example 3** Write an equation for the \( n \)th term of each arithmetic sequence. Then graph the first five terms of the sequence.
  
  1. 15, 13, 11, \ldots
  2. -2, 1, 8, \ldots

- **Example 4** **SAVINGS** Kasia has $552 in a savings account. After one month, she has $580 in the account. The next month, the balance is $603. The balance after the third month is $660. Write a function to represent the arithmetic sequence. Then graph the function.

---

**Practice and Problem Solving**

**Extra Practice on page 195**

- **Example 1** Determine whether each sequence is an arithmetic sequence. Write yes or no.
  
  1. -3, 5, 9, \ldots
  2. -10, -7, -4, \ldots
  
  Explain.

- **Example 2** Find the next three terms of each arithmetic sequence.
  
  1. 0.02, 0.08, 0.14, 0.20, \ldots
  2. 2, 4, 6, 8, \ldots

- **Example 3** Write an equation for the \( n \)th term of the arithmetic sequence. Then graph the first five terms in the sequence.
  
  1. 10, 8, 6, 4, \ldots
  2. 0, -2, -4, \ldots

- **Example 4** **AMUSEMENT PARKS** Skuboi and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.
  
  a. Write a function to represent the arithmetic sequence.
  
  b. Graph the function and determine the domain.

---

**23. MODELING** The table shows how Ryan is paid at his lumber yard job.

<table>
<thead>
<tr>
<th>Linear Feet of 2 x 4 Planks Cut</th>
<th>Amount Paid in Commission ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>30</td>
<td>24</td>
</tr>
<tr>
<td>40</td>
<td>32</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>60</td>
<td>48</td>
</tr>
<tr>
<td>70</td>
<td>56</td>
</tr>
</tbody>
</table>

- **a.** Write a function to represent Ryan's commission.
- **b.** Graph the function and determine the domain.
24. The graph is a representation of an arithmetic sequence.
   a. List the first five terms.
   b. Write the formula for the nth term.
   c. Write the function.

25. **Newspapers** A local newspaper charges by the number of words for advertising. Write a function to represent the advertising cost.

26. The fourth term of an arithmetic sequence is 8. If the common difference is 2, what is the first term?

27. The common difference of an arithmetic sequence is -5. If a_{12} is 22, what is a_1?

28. The first four terms of an arithmetic sequence are 28, 20, 12, and 4. Which term of the sequence is 36?

29. **Cars** Jamal’s odometer of his car reads 24,521. If Jamal drives 45 miles every day, what will the odometer reading be in 25 days?

30. **Yearbooks** The yearbook staff is unpacking a box of school yearbooks. The arithmetic sequence 281, 270, 259, 248, ... represents the total number of ounces that the box weighs as each yearbook is taken out of the box.
   a. Write a function to represent this sequence.
   b. Determine the weight of each yearbook.
   c. If the box weighs at least 11 ounces empty and 292 ounces when it is full, how many yearbooks were in the box?

31. **Sports** To train for an upcoming marathon, Olivia plans to run 3 miles per day for the first week and then increase the daily distance by a half mile each of the following weeks.
   a. Write an equation to represent the nth term of the sequence.
   b. If the pattern continues, during which week will she run 10 miles per day?
   c. Is it reasonable to think that this pattern will continue indefinitely? Explain.

---

**H.O.T. Problems** Use Higher-Order Thinking Skills

32. **Open Ended** Create an arithmetic sequence with a common difference of -10.

33. **perseverance** Find the value of x that makes x + 8, 4x + 6, and 3x the first three terms of an arithmetic sequence.

34. **Reasoning** Compare and contrast the domain and range of the linear functions described by 3x + 2y = 5 and a_{n+1} = a_n + (n - 1)d.

35. **Challenge** Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain. If yes, find the common difference and the next three terms.
   a. 2x + 1, 3x + 1, 4x + 1, ...
   b. 2x, 4x, 8x, ...

36. **Writing in Math** How are graphs of arithmetic sequences and linear functions similar? Different?

---

**Spiral Review**

**Standardized Test Practice**

37. **Gridded Response** The population of Westerville is about 35,000. Each year the population increases by about 400. This can be represented by the following equation, where n represents the number of years from now and p represents the population.

\[ p = 35,000 + 400n \]

In how many years will the Westerville population be about 38,200?

38. Which relation is a function?
   a. \( \{(5, 6), (4, -3), (2, -1), (4, 2)\} \)
   b. \( \{(3, -1), (-3, 5), (3, 4), (3, 6)\} \)
   c. \( \{(-2, 3), (0, 2), (-2, -1), (-1, 3)\} \)
   d. \( \{(-5, 6), (4, -3), (2, -1), (0, 2)\} \)

39. Find the formula for the nth term of the arithmetic sequence.

\[ a_n = 3n - 4 \]

\[ a_n = -7n + 10 \]

\[ a_n = 3n - 10 \]

\[ a_n = -7n + 3 \]

40. **Statistics** A class received the following scores on the ACT. What is the difference between the median and the mode in the scores?

\[ \begin{align*}
   18, & 26, 20, 30, 25, 21, 32, 19, 22, 29, 27, 24 \\
\end{align*} \]

   A 1 \hspace{1cm} C 3 \hspace{1cm} B 2 \hspace{1cm} D 4

---

**Find the slope of the line that passes through each pair of points.** (Lesson 3-3)

41.
42.

**Find the slope of the line that passes through each pair of points.** (Lesson 3-3)

43. (3, 3), (2, 6)
44. (9, 2), (3, -1)
45. (2, 8), (-2, -4)

**Solve each equation. Check your solution.** (Lesson 2-8)

46. \( 5x + 7 = -8 \)
47. \( 8 = 2 + 3n \)
48. \( 12 = \frac{c - 6}{2} \)

**Skills Review**

**Graph each point on the same coordinate plane.**

50. A(2, 5) \hspace{1cm} 51. B(-2, 1) \hspace{1cm} 52. C(-3, -1)

53. D(0, 4) \hspace{1cm} 54. F(5, -3) \hspace{1cm} 55. G(-5, 0)