Def. Proposition: a statement that is true or false.
Example: $2+2=4$ (true)
Example: $2+2=5$ (false)

Example: For every non-negative number $n$, $\sqrt{n}$ is prime, if only divisible by 1 and itself.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\sqrt{n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

Def. Proposition: a statement that depends on one or more variables.

Example: $n$ is a perfect square

*Proof.* If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

\[ \frac{df}{dx} = 2x + 1, \quad \frac{d}{dx} \left( x^2 + x + 1 \right) = 2x + 1 \]

Thus, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

\[ \text{Def. Implication: a statement of the form } P \implies Q. \]

Direct Proof: A sequence of implications, each of which do not have to be proved, that combine to form its own implication.

*Proof.* If $P$, then $P_1$. Therefore, $P$. Therefore, $P_2$.

\[ P \implies (P_1 \implies P_2) \]

\[ \text{Thus, } -x^2 + x + 1 \leq 0. \]

Therefore, $-x^2 + x + 1 \leq 0$.

Theorem: If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

Prop. If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

Thus, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Deduction: a proposition that depends on one or more variables.

Example: $n$ is a perfect square

*Proof.* If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

Thus, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Deduction: a proposition that depends on one or more variables.

Example: $n$ is a perfect square

*Proof.* If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

Thus, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Deduction: a proposition that depends on one or more variables.

Example: $n$ is a perfect square

*Proof.* If $0 \leq x \leq 2$, then $f(x) = x^2 + x + 1 \geq 0$.

Thus, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Therefore, $-x^2 + x + 1 \leq 0$.

Ex.: Assume proposition accepted as being true

\[ \text{does not need to be proved.} \]

This does up to your best judgment.