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Planning Optimal Grasps

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Abstract

In this paper we will address the problem of planning optimal grasps. Two general optimality criteria, that consider the total finger force and the maximum finger force will be introduced and discussed. Moreover their formalization, using various metrics on a space of generalized forces, will be detailed. The geometric interpretation of the two criteria will lead to an efficient planning algorithm. An example of its use in a robotic environment equipped with two-jaw and three-jaw grippers will also be shown.

1 Introduction

Planning a good grasp is fundamental in applications that require the objects to be firmly held by the robot. In this paper we aim to introduce and discuss the formalization of *quality criteria*, that can be used to judge how good is a given grasp configuration. One of our criteria is new, and the other is the same as that proposed in [7]. We give physical motivation for both that derive from consideration of limits on the finger actuators. We give a geometric interpretation of the criteria which unifies them, and allows simple algorithms for optimal grasp planning according to either criterion. The criteria themselves are very general, and apply to any kind of mechanism (grippers, multifingered hand, cooperative robot arms, and so on). The geometrical aspects of grasping will be emphasized while the problem of controlling compliance between the object and the jaws is not considered.

Flexibility and efficiency are both major requirements in robotic applications, and robotics research needs to focus on best resolving this conflict. As clearly pointed out in [1] the flexibility-efficiency issue stands at the core of robotics.

In the past there has been a growing interest towards multifingered hands, because it was believed that their extreme flexibility could enhance the performances of assembly systems. Because of their intricate design, they are difficult to control and plan for, expensive, unreliable, and require much computing power. A different paradigm (RISC robotics) suggest using different grippers each of which is suitable for a small subset of operations. Those grippers can be interchanged on the same robot, or they can be mounted on different robots (or modules) and work in cooperation. In this approach, one must decide which of the grippers should be used for handling a given part.

In choosing among different grippers, the planner uses quality criteria for deciding which is the best solution for each gripper and the best overall. Our criteria can be calculated easily for a wide variety of gripper and part types, although the implementation so far has been for planar objects.

The paper follows this path: in section two and three we will summarize the major working hypothesis and definitions that underlie this work. In section four, we introduce and discuss the quality criteria we are proposing. In section five we will present the algorithm for grasp planning and we figure out its complexity.

2 Working hypotheses

Gripper jaws can exert forces and torques on the grasped objects through the contact points. Given the position of the gripper and the object to be grasped, how can we say "this is a good grasp" ? One idea is formally represented by the definition of "force closure grasp" [4]. A grasp is said to be force closure if it is possible to apply forces and torques at the contact points such that any external force and torque can be balanced. In a force closure grasp, finger locations do not change to counter external forces. It the following we will only consider grasp configurations that a fortiori satisfy the force closure condition. In fact our criteria make quantitative the notion of good force closure grasp. Both criteria must be positive for any force closure grasp.

A key point in modeling a grasp is the definition of the contact between objects and fingers. We will use the "hard-finger" model. In this model, fingers can exert any force pointing into the friction cone at the point of contact. If the contact is more complex (i.e. it is not a point contact, but it is an edge contact or a face contact) it can be described as the convex sum of proper point contacts [4]. Edge contact is equivalent to two point contacts located at the end of the edge,

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while face contacts are equivalent to point contacts at the vertices of the face.

Forces and torques acting on an object can be represented in a 6-dimensional space, having three variables for the three components of the total moment acting on the object and three more for the total force. This space is called "the wrench space". With 2-dimensional objects wrench space is 3-dimensional consisting of two orthogonal force directions, lying in the plane of the object and the torque perpendicular to the force plane. Any force and torque on the object can be represented by a point in the wrench space. Hence we have an immediate representation of each point contact force exerted by the fingers. For each point-to-edge contact we have one primitive contact and two primitive wrenches that describe it. For each edge-to-edge contact we have two primitive contacts and therefore four primitive wrenches.

3 Related work

In [6], Markenscoff and Papadimitriou studied optimum *form semiclosure grips*, i.e., grips that can balance any force through the centre of gravity of the object by finger forces. They emphasized the use of a quality criterion that minimizes the sum of the intensities of the applied forces (also suggested in [5]) although, as they observed, their formalization is suitable for being used also for minimizing the worst-case maximum of the finger forces. The solution proposed in this paper is more general, and unifies in a general framework the formalization of the optimality criteria. We will consider *force-closure* grasps, i.e., grasps that can resist any external wrench. Moreover the proposed solution can be applied to any 3-dimensional object, even if we only show examples of its use on 2-dimensional ones.

In [7], grasps are evaluated using the *Quantitative Steinitz's Theorem*. The "efficiency" of a grasp is given as the value of the radius of the largest closed ball, centered in the origin of the wrench space, contained in the set of all the possible wrenches that can be resisted by appling at most unit forces at the contact points. These is one of the two measures we study in the following.

4 The Quality of Grasp

Some grasp configurations can be better than others in the sense that they can balance every external force, without applying too large finger forces. Avoiding large forces minimizes the deformation of both the object and the jaws. Moreover, it minimizes the power for actuating the gripper. An intuitive way of judging the quality of a grasp is to consider the ratio between the magnitude of the maximum wrench to be resisted (over all the possible directions), and some notion of the magnitude of the applied finger forces.

This concept needs to be formalized, in order to be effectively used in a grasp planner. First of all we need to make clear what the *magnitude* of a wrench is, in this section. Then a precise definition for *magnitude of the applied forces* will be given in the next.

In section 2, we mentioned how forces and torques can be represented by points in the wrench space. In general, a wrench is a vector $\boldsymbol{w} \in \mathbf{R}^p$, defined as follows:

$$\boldsymbol{w} = \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{\tau} \end{pmatrix}$$

where $\boldsymbol{F} \in \mathbf{R}^3$ (or $\mathbf{R}^2$) is the force vector and $\boldsymbol{\tau} \in \mathbf{R}^3$ (or $\mathbf{R}$) is the torque vector, acting on the object. The dimension p is usually 6, but it reduces to 3, while considering 2-dimensional objects. Let's denote as $\mathcal{W}$ the wrench space.

We define the magnitude of a wrench, (and we indicate as $\|\boldsymbol{w}\|$) the following quantity:

$$\|\boldsymbol{w}\| = \sqrt{\|\boldsymbol{F}\|^2 + \lambda\|\boldsymbol{\tau}\|^2}$$

The choice of λ is somehow arbitrary, because the torque magnitude can be indipendently scaled with respect to the force magnitude. It should be remembered that forces and torques are dimensionally different. Choosing a value for λ equal to 1 means measuring the $\|\boldsymbol{w}\|$ according an L_2 metric.

4.1 Representing finger forces

Let's denote the force exerted by the finger at the i-th point contact as $\boldsymbol{f}_i$. Because of friction, this force is in the friction cone, and it is a positive combination of forces along the extrema of the friction cone itself. Let's define $f_i^{\perp}$ the component of this force along the normal to the object surface at the contact point, and f_i^t the component tangent to the object surface at the point contact. Coulomb's law says that $f_i^{\perp} \geq \mu f_i^t$, with μ the coefficient of friction.

In the planar case, for each contact, we have:

$$\boldsymbol{f}_i = \beta_l \hat{\boldsymbol{f}}_{i,l} + \beta_r \hat{\boldsymbol{f}}_{i,r}$$

where $\beta_l, \beta_r \geq 0$, and $\hat{\boldsymbol{f}}_{i,l}, \hat{\boldsymbol{f}}_{i,r}$ are unit vectors along the two lines delimiting the friction cone.

In the 3-dimensional case the actual friction cone can be approximated with a proper pyramid. We have:

$$\boldsymbol{f}_i \approx \sum_{h=1}^{m} \beta_h \hat{\boldsymbol{f}}_{i,h}$$

where $\beta_h \geq 0$, and $\hat{\boldsymbol{f}}_{i,h}$ are unit vectors on the friction cone surface.

The above expressions can be rewritten in terms of $f_i^{\perp}$. In general $\boldsymbol{f}_i$ is given by a convex combination of forces along the extrema of the friction cone, whose normal component is $f_i^{\perp}$. We have, in the planar case:

$$\boldsymbol{f}_i = \alpha_l \boldsymbol{f}_{i,l} + \alpha_r \boldsymbol{f}_{i,r}$$

with $\alpha_l, \alpha r \geq 0, \alpha_l + \alpha_r = 1$.

In 3-D we have:

$$\boldsymbol{f}_i \approx \sum_{h=1}^{m} \alpha_h \boldsymbol{f}_{i,h}$$

where $\alpha_h \geq 0$ and $\sum_{h=1}^{m} \alpha_h = 1$.

We should notice that knowledge of the normal contact force is not enough for specifying the actual contact force. In fact there can be infinitely many contact forces (inside the friction cone) that satisfy the conditions above and have the same normal magnitude.

Let be $\boldsymbol{g}$ the generalized force vector, built by piling up all the $f_i^{\perp}$. Given n contacts, we have the following definition:

$$\boldsymbol{g} = \begin{pmatrix} f_1^{\perp} \\ \vdots \\ f_n^{\perp} \end{pmatrix}$$

As we pointed out earlier, specifying $\boldsymbol{g}$ does not determine the actual wrench acting on the object (or equivalently of the wrench that can be resisted by applying $\boldsymbol{g}$).

Hence it is worthwhile defining a predicate $A : \mathcal{W} \times \mathcal{G} \rightarrow \{T, F\}$, where $\mathcal{G}$ is the set of all possible generalized forces $\boldsymbol{g}$. For each $\boldsymbol{w} \in \mathcal{W}$ and $\boldsymbol{g} \in \mathcal{G}$, we say that $\boldsymbol{w}A\boldsymbol{g}$ is true if the wrench $\boldsymbol{w}$ belongs to the set of wrenches that can be resisted through the generalized force $\boldsymbol{g}$.

We define the set A as the set of all couples $(\boldsymbol{w}, \boldsymbol{g})$ such that $\boldsymbol{w}A\boldsymbol{g}$ is true, i.e.,

$$A = \{\ (\boldsymbol{w}, \boldsymbol{g}) \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true}\}$$

Moreover we can define as $\boldsymbol{w}A$ the set of generalized forces that can resist the wrench $\boldsymbol{w}$, and as $A\boldsymbol{g}$ the set of wrenches that can be resisted by $\boldsymbol{g}$. We have:

$$\boldsymbol{w}A = \{\ \boldsymbol{g} \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true }\}$$

and

$$A\boldsymbol{g} = \{\ \boldsymbol{w} \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true}\}$$

4.2 Grasp quality measure

Let's define as *local grasp quality measure* (LQ) the following quantity:

$$LQ_{\boldsymbol{w}} = \max_{\boldsymbol{g} \in \boldsymbol{w}\mathrm{A}} \frac{||\boldsymbol{w}||}{||\boldsymbol{g}||}$$

which is the best ratio between resulting wrench and applied force for a given wrench direction $\mathbf{w}$.

With this definition, we are ready now to introduce the *grasp quality measure* (Q). Given a grasp configuration (i.e. a set of point contacts on the object), Q is defined as follows:

$$Q = \min_{\boldsymbol{w}} LQ_{\boldsymbol{w}}$$

We take the minimum because we usually have no control over the wrench that the gripper must resist. We therefore want to guarantee a level of performance as judged by the local quality measure over all possible wrenches, and this is the measure Q

Notice that for a given direction of $\mathbf{w}$, the value of Q is not dependent on $||\boldsymbol{w}||$, because we are considering the ratio of wrenches and applied forces, and both scale linearly with each other. Because of this invariance of Q with scaling, minimizing over $\boldsymbol{w}$ means minimizing over the directions of $\boldsymbol{w}$. We have:

$$Q = \min_{\hat{\boldsymbol{w}}} LQ_{\hat{\boldsymbol{w}}}$$

and $\hat{\boldsymbol{w}} = dir(\boldsymbol{w}) = \boldsymbol{w}/||\boldsymbol{w}||$. Without loss of generality, we choose $||\mathbf{w}||$ so that $||\boldsymbol{g}|| = 1$. Let's define the set $B \subset A$ as follows:

$$B = \{\ (\boldsymbol{w}, \boldsymbol{g}) \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true, and } ||\boldsymbol{g}|| = 1\}$$

Equivalently, we define a predicate $B : \mathcal{W} \times \mathcal{G} \rightarrow \{T, F\}$, such that $\boldsymbol{w}B\boldsymbol{g}$ is true if the wrench $\boldsymbol{w} \in A\boldsymbol{g}$ and $||\boldsymbol{g}|| = 1$.

We have:

$$\boldsymbol{w}B = \{\ \boldsymbol{g} \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true, and } ||\boldsymbol{g}|| = 1\}$$

and

$$B\boldsymbol{g} = \{\ \boldsymbol{w} \mid \boldsymbol{w}A\boldsymbol{g} \text{ is true, and } ||\boldsymbol{g}|| = 1\}$$

Hence we can rewrite LQ, as follows:

$$LQ_{\hat{\boldsymbol{w}}} = \max_{\boldsymbol{w} \in B\mathcal{G}} ||\boldsymbol{w}||$$

We denote with $B\mathcal{G}$ the set that is the union of all the sets $B\boldsymbol{g}$, i.e.:

$$B\mathcal{G} = \bigcup_{\boldsymbol{g} \in \mathcal{G}} B\boldsymbol{g}$$

Taking the maximum value of the wrench module for each wrench that belongs to $B\mathcal{G}$, means considering those wrenches that are on the boundary of $B\mathcal{G}$. Hence:

$$LQ_{\hat{\boldsymbol{w}}} = \{\ ||\boldsymbol{w}|| \mid \boldsymbol{w} \in Bd(B\mathcal{G})\ \}$$

Finally, we can rewrite Q as follows:

$$Q = \min_{\hat{\boldsymbol{w}}} LQ_{\hat{\boldsymbol{w}}} = \min_{\hat{\boldsymbol{w}}} \{\ ||\boldsymbol{w}|| \mid \boldsymbol{w} \in Bd(B\mathcal{G})\ \}$$

As for the force-closure condition, there is a simple geometrical interpretation of Q. The force-closure condition is equivalent to having the origin of the wrench-space contained in the convex hull of the primitive wrenches [2].

Similarly, the proposed quality criteria can be easily interpreted in the wrench space, leading to a geometrical analysis of the grasp quality. The geometrical interpretation of the last formula is: taking the maximum of wrenches in $B\mathcal{G}$, means getting the boundary of $B\mathcal{G}$. Then, Q is just the distance of the nearest point to the origin, from the origin itself. That is:

$$Q = \min_{\boldsymbol{w} \in \mathrm{Bd}(B\mathcal{G})} ||\boldsymbol{w}||$$

in other words, Q is the radius of the largest sphere (centered at the origin) which is contained in $B\mathcal{G}$.

Of course, there can still be some directions where the reaction wrench can be greater, but we want to be assured we get a lower bound over all directions. Hence the grasp quality is equal to the magnitude of the minimum, over all wrench directions, of the maximum wrench we can exert in that direction.

In the above definition we postponed the definition of $||\boldsymbol{g}||$. In fact, different definitions can represent different quality criteria. We are proposing two different criteria for evaluating the quality of a grasp. The first is concerned with finding the grasp configurations that maximize the wrench, given independent force limits, i.e. that minimize the worst-case force applied at any point contact. The second criterion minimizes the sum of all the applied forces: because the magnitude of the force is proportional to the total current in motors and amplifiers, using this criteria will result in the minimization of the power need to actuate the gripper.

These two criteria can be represented by using different metrics in the definition of $||g||$. In particular, using an L_∞ metric ($||g|| = \max(g_1, ..., g_n)$), we can represent the former criteria, while using an L_1 metric ($||g|| = (g_1 + ... + g_n)$) we can represent the latter. The two criteria will be discussed in the next two subsections.

Finally, it should be pointed out that these are general criteria, that can be applied with any generic mechanism performing a grasp operation. Even though we are considering two and three jaw grippers, the above criteria can be evaluated for any kind of multifingered hand as well as cooperative arms.

4.3 Minimizing the maximum finger force

While considering a grasp configuration that is optimal with respect to the maximum applied finger force, it is reasonable to state the hypothesis that the applied forces are individually and independently upper-bounded. Moreover we can say that the upper bound can be considered equal to 1. This is because we are considering the ratio of wrenches and applied forces, and both scale linearly with each other. Thus we set $||g|| = 1$, using an L_∞ metric. Moreover, the reaction forces are contained in the friction cone and they can be represented by some convex combination of vectors along the boundary of the friction cone.

Considering the force at the i-th contact we have:

$$\boldsymbol{f}_i = \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{f}_{i,j}$$

with $\alpha_{i,j} \geq 0$ and $\sum_{j=1}^{m} \alpha_{i,j} \leq 1$. The $\boldsymbol{f}_{i,j}$ are the vectors that generate the friction cone.

The reaction torque $\boldsymbol{\tau}_i$ is given by $\boldsymbol{r}_i \times \boldsymbol{f}_i$, where $\boldsymbol{r}_i$ is the vector pointing from the center of mass of the object to the point contact where the force is applied. We have:

$$\boldsymbol{\tau}_i = \sum_{j=1}^{m} \alpha_{i,j} (\boldsymbol{r}_i \times \boldsymbol{f}_{i,j})$$

Using the wrench notation we can say:

$$\boldsymbol{w}_i = \sum_{i=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j}$$

and the set of all the possible wrenches originating from the contact i can be denoted as:

$$W_i = \{\boldsymbol{w}_i \mid \boldsymbol{w}_i = \sum_{i=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j}, \ \ \alpha_{i,j} \geq 0, \ \sum_{j=1}^{m} \alpha_{i,j} \leq 1\}$$

This can be done for each contact. The total wrench acting on the object is :

$$\boldsymbol{w} = \sum_{i=1}^{n} \boldsymbol{w}_i$$

and the set of all the possible wrenches acting on the object is given by:

$$\mathcal{BG} \ = \ W_{L_\infty} = W_1 \oplus \ldots \oplus W_n$$

The last formula says that the total wrench exerted on the object belongs to the set that is the Minkowski sum of the convex sets that correspond to the contacts. Hence we have a geometric representation of the set that correspond to $\mathcal{BG}$. Because we can exchange the Minkowski sum with the ConvexHull operation we have:

$$W_{L_\infty} = ConvexHull \ (\bigoplus_{i=1}^{n} \{\boldsymbol{w}_{i,1}, \ldots \boldsymbol{w}_{i,m}\})$$

This last formula gives an efficient way to compute W_{L_∞}, starting from the primitive wrenches that describe each contact. The Minkowski sum over a finite number of sets with a finite number of elements gives a set that is finite. Hence it is enough to compute the convex hull over the elements of that set. The quality measure (Q_∞) is the distance of the nearest facet of the convex hull, from the origin.

4.4 Minimizing the total finger force

In this case we state the hypothesis that the sum of the magnitude of the forces at the contact points is upper-bounded, and we take the upper bound to be 1. This is equivalent to say that $||g|| = 1$, over an L_1 metric. We have that the total force is:

$$\boldsymbol{f} = \sum_{i=1}^{n} \boldsymbol{f}_i$$

Every $\boldsymbol{f}_i$ is in the friction cone and it is a convex combination of some vectors along the extrema of the friction cone. Hence $\boldsymbol{f}$ can be rewritten as:

$$\boldsymbol{f} = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{f}_{i,j}$$

and $\alpha_{i,j} \geq 0, \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} \leq 1$

Similarly we have that the total wrench acting on the object can be expressed by:

$$\boldsymbol{w} = \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j}$$

and the set of all the possible wrenches is:

$$W_{L_1} = ConvexHull(\bigcup_{i=1}^{n} \{\mathbf{w}_{i,1}, \ldots, \mathbf{w}_{i,m}\}$$

The last formula says that the total wrench exerted on the object belongs to the set that is the convex hull generated by the wrenches that correspond to the primitive contacts on the object. Again, the formula gives a way to compute W_{L_1}, by computing the convex hull over a finite set of points. The quality measure (Q_1) is the distance of the nearest facet of the convex hull, from the origin.

The two methods are somehow related. In fact, $W_{L_\infty} \supseteq W_{L_1}$, and $Q_\infty \geq Q_1$, because we are computing the convex hull starting from two sets, such that one is a subset of the other. Anyway, in general, the two criteria are not equivalent. Given two different grasp configurations A and B with their quality measures, Q_∞^A,Q_1^A, Q_∞^B,Q_1^B, with, for instance, $Q_\infty^A > Q_\infty^B$, nothing can be said about the relation between Q_1^A and Q_1^B. Hence a grasp configuration that is optimal according one criterion can be non-optimal according the other criterion.

Finally, an example of the geometrical representation of the proposed quality criteria is given in figure 1, for the grasp of a triangle by a 3-jaw gripper (1-a). In 1-b, the primitive wrenches of the three point contacts are shown, while in 1-c the shadow areas represent the wrenches originating from each contact. In 1-d and 1-e W_{L_∞} and W_{L_1} are respectively shown.

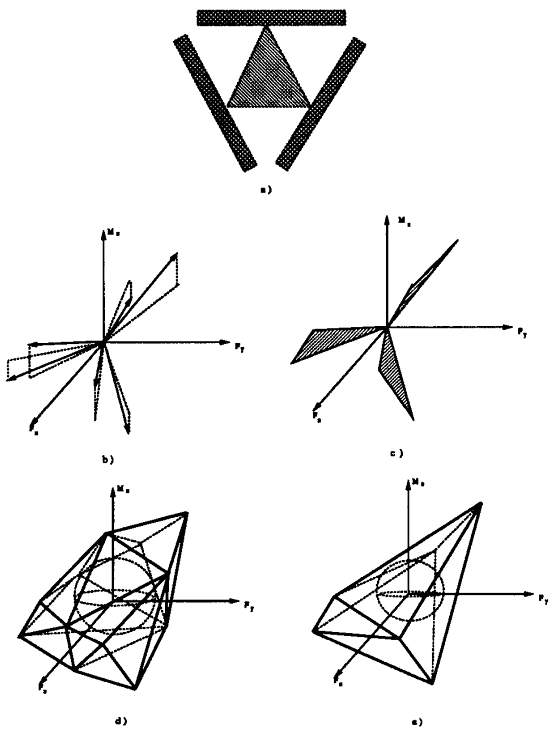

Figure 1: Graphic Evaluation of the Quality Criteria

5 An Example of Using the Quality Criteria

In the next subsections, we will present an algorithm that can evaluate the quality of given grasps. In the following we will use the criterion for minimizing the total force exerted on the object. At first the algorithm will be described for two-jaw gripper grasping polygonal objects, then the extension for considering three-jaw grippers will be discussed.

5.1 Two-jaw gripper grasping a polygonal object

In this section we analyze the quality of grasp performed on polygonal objects by a two-jaw parallel gripper. The dimension of the edges of the polygon have side length comparable to the width of the finger. Let's consider the case in which we have one side of the polygon that completely touches one jaw. We will call this type of contact a side-to-side contact. Given a two-jaw gripper the other jaw can be in contact either with a vertex, or a side, depending on the shape and symmetries of the given polygon. The number of possible configuration grows linearly as the number of edges of the polygons The planning algorithms can be summarized as follows:

- Given a side of the polygon compute the farthest vertex with respect to this side. Call them respectively the "opponent" and the "base". If there are two vertices that satisfy this condition then consider the side between them as opponent.
- Determine the position of the primitive contacts both of the base and the opponent. Represent for each type of contact its primitive wrenches in the wrench-space.
- Compute the convex hull and determine the facet of minimum distance from the origin.

This algorithm has to be repeated for each side of the polygon comparing at the end of each step the new minimum with the previous one and keeping track of the grasp configuration with the larger minimum. The number of possible configuration grows linearly as the number of edges of the polygons, while the computation of the convex hull of the primitive wrenches in the wrench space, takes constant time. Hence the complexity of this algorithm is $O(n)$.

We have also to consider the case in which there are two vertex-to-side contacts with the jaws. It is not difficult to see that if two vertex-side contacts are specified, the largest sphere volume is attained when the line between the contacts is at right angles to the jaws. Thus there are only a finite number of part configurations to consider in selecting a best grasp.

5.2 Using a three-jaw gripper

We analyze the quality of grasp performed on polygonal objects by a three-jaw gripper. Again let's start considering convex objects.

Most of the considerations above are still valid in the three-fingered case: the main difference is in the procedure for determining the primitive contacts, because the geometry of the gripper is changed. First of all consider the case where one of the sides is totally in contact with one of the fingers. To determine the type of contact involving the other two fingers, we have to consider the orientations of the sides of the polygon and of the triangle formed by the fingers. Considering a counterclockwise representation of the polygons, we can say that there will be a point-to-side contact between the polygon and one of the jaws, and such contact will involve the vertex that belongs to two sides such that the former has orientation lesser than the corresponding finger-triangle side orientation, and the latter has orientation greater than the corresponding finger-triangle side orientation (see fig. 2). If one of this orientation is equal to the finger triangle orientation we have a side-to-side contact.

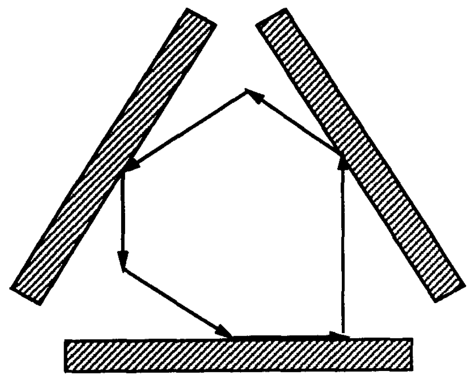

Figure 2: Three-jaw Gripper grasping a Polygonal Object

In the case of a three fingered gripper there is an additional test in order to avoid collision among the fingers. It is necessary to check if there is any vertex of the polygons that lies out of the triangle formed by the finger when they are completely closed. If the result of the test is negative, no grasp can be performed. Let's summarize the major point of the grasping algorithm:

- Given a side of the polygon (the "base") compute the elements that are in contact with the other two jaws using the previous considerations. Call them respectively the "right opponent" and the "left opponent".
- Determine the position of the primitive contacts both of the base and the two opponents. Represent for each type of contact its primitive wrenches in the wrench-space.
- Compute the convex hull and determine the facet of minimum distance from the origin.

This algorithm has to be repeated for each side of the polygon comparing at the end of each step the new minimum with the previous one and keeping track of grasp configuration with the larger minimum.

We have also to consider the case in which the polygon has three vertex-to-side contacts with the jaws. The three vertices should form a triangle such that it can touch the triangle formed by the jaws. By enumerating all the possible triangle generated by the given polygon we can find the possible candidate sticking configuration. Hence, the algorithm should be applied to these configuration, to evaluate the quality of the grasp.

Again, if the polygon is not convex, we can reason on its convex hull. Hence, we can apply the previous algorithm to this new polygon, if the condition about the dimensions of the jaws and the magnitude of the length of the edges of the convex hull of the given polygon, is satisfied.

6 Conclusion

In this paper we have formalized two quality criteria, for planning optimal force-closure grasps. Their geometrical interpretation in the wrench space has been emphasized, showing how a fast planning algorithm can be derived.

Further work is required to devise an algorithm that uses the optimality criteria when there exist an infinity of candidate grasps configurations. This is the case we have without the hypothesis that the dimensions of the jaws are bigger than the dimension of the object. Moreover, as also pointed out in [7], the torque and force dimensions are non-comparable. Hence one could propose a different definition of optimum wrench, i.e a different definition for $||\boldsymbol{w}||$. Rather than seeking the largest *sphere* contained in $\mathcal{BG}$, we would then look for the largest *ball* under the $||\cdot||$ measure.

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