Solving Inequalities by Multiplication and Division

Then
- You solved equations by using multiplication and division.

Now
- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Why?
- Terril received a gift card for 839 of music downloads. If each download costs $0.99, the number of downloads he can purchase can be represented by the inequality 0.99x ≤ 80.

1. **Solve Inequalities by Multiplication**
   - If you multiply each side of an inequality by a positive number, the inequality remains true.
   - \( 4 > 2 \)  \( \times 3 \)  \( 12 > 6 \)
   - \( 4 \times 3 \)  \( 12 \)
   - The direction of the inequality remains the same.

2. **Solve Inequalities by Division**
   - If you multiply each side of an inequality by a negative number, the inequality symbol changes direction.
   - \( 7 < 9 \)  \( \div (-2) \)  \( -3.5 < -4.5 \)
   - \( -14 > -18 \)

These examples demonstrate the **Multiplication Property of Inequalities**.

### Key Concept: Multiplication Property of Inequalities

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<th>Words</th>
<th>Symbols</th>
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<td>If both sides of an inequality that is true are multiplied by a positive number, the resulting inequality is also true.</td>
<td>( a &gt; b )  ( \times c )  ( ac &gt; bc )</td>
<td>( 6 &gt; 3 )  ( 6 \times 2 &gt; 3 \times 2 )  ( 12 &gt; 6 ) ( 2.1 &lt; 5 )  ( 2.1 \times 0.5 &lt; 5 \times 0.5 )  ( 1.05 &lt; 2.5 )</td>
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- Solve for \( r \).
  - \( \frac{2}{3}r < 84 \)
  - \( r < 126 \)
  - \( r < 84 \)

- **Real-World Example 1**: Write and Solve an Inequality
  - **Survey**: Of the students surveyed at Madison High School, fewer than eighty-four said they have never purchased an item online. This is about one eighth of those surveyed. How many students were surveyed?
  - **Understand**: You know the number of students who have never purchased an item online and the portion this is of the number of students surveyed.
  - **Plan**: Let \( n \) be the number of students surveyed. Write an open sentence that represents this situation.
  - **Words**: One eighth times the number of students surveyed is less than 84.
  - **Symbols**: \( \frac{1}{8}n < 84 \)
  - \( n < 672 \)
  - \( \frac{1}{8}(672) \leq 84 \)

### Guided Practice

1. **BIOLOGY** Mount Kinabalu in Malaysia has the greatest concentration of wild orchids on Earth. It contains more than 750 species, or about one fourth of all orchid species in Malaysia. How many orchid species are there in Malaysia?

   - **Words**: More than 30,000 different orchids species live in the wild on every continent except Antarctica.
   - **Symbols**: \( x > 30000 \)
   - \( x > 7500 \)

   - You can also use multiplicative inverses with the Multiplication Property of Inequalities to solve an inequality.

### Example 2: Solve by Multiplying

- **Solve** \( \frac{2}{3}r < 21 \). Graph the solution on a number line.
  - **Words**: The solution set is \( r > 49 \).
  - **Symbols**: \( r > 49 \)

- **Guided Practice**
  - **Example**: \( \frac{2}{3}y > 10 \)
  - **Example**: \( \frac{1}{3}m = -3 \)
  - **Example**: \( \frac{3}{2} < 5 \)
Solve Inequalities by Division

If you divide each side of an inequality by a positive number, then the inequality remains true.

\[ \frac{-10}{2} < \frac{-5}{2} \]

Simplify.

\[ -5 < -\frac{5}{2} \]

Notice that the direction of the inequality remains the same. If you divide each side of an inequality by a negative number, the inequality symbol changes direction.

\[ \frac{15}{-3} > \frac{18}{-3} \]

Simplify.

\[ -5 > -6 \]

These examples demonstrate the Division Property of Inequalities.

Key Concept: Division Property of Inequalities

If both sides of a true inequality are divided by a positive number, the resulting inequality is also true.

- For any real numbers \( a \) and \( b \) and any positive real number \( c \), if \( a > b \), then \( \frac{a}{c} > \frac{b}{c} \).
- And, if \( a < b \), then \( \frac{a}{c} < \frac{b}{c} \).

If both sides of a true inequality are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.

- For any real numbers \( a \) and \( b \) and any negative real number \( c \), if \( a > b \), then \( \frac{a}{c} < \frac{b}{c} \).
- And, if \( a < b \), then \( \frac{a}{c} > \frac{b}{c} \).

Example 3: Divide to Solve an Inequality

Solve each inequality. Graph the solution on a number line.

a. \( 60t > 8 \)

Divide each side by 60.

\[ t > \frac{2}{15} \]

Simplify.

b. \( -7d \leq 147 \)

Divide each side by \(-7\).

\[ d \geq -21 \]

Simplify.

Guided Practice

3A. \( 8p < 56 \)

3B. \( -42 > 6r \)

3C. \( -12r > 15 \)

3D. \( -\frac{1}{2}c < 6 \)
Match each inequality to the graph of its solution.

35. \( \frac{-3}{4}x \leq 9 \)
36. \( 25y \geq 8 \)
37. \( 3.6p < -4.5 \)
38. \( 2.3 < -5t \)

\( a. \) \[ \quad \]
\( b. \) \[ \quad \]
\( c. \) \[ \quad \]
\( d. \) \[ \quad \]

\( 20 \) CANDY Fewer than 42 employees at a factory stated that they preferred fudge over fruit candy. This is about two thirds of the employees. How many employees are there?

40. TRAVEL A certain travel agency employs more than 275 people at all of its branches. Approximately three fifths of all the people are employed at the west branch. How many people work at the west branch?

41. MULTIPLE REPRESENTATIONS The equation for the volume of a pyramid is \( V = \frac{1}{3}Bh \) where \( B \) is the area of the base times the height.

\( a. \) Geometric Draw a pyramid with a square base \( 6 \) cm long and a height of \( 9 \) cm.

\( b. \) Numerical Suppose the pyramid has a volume of \( 72 \) cm\(^3\). Write an equation to find the height.

\( c. \) Tabular Create a table showing the value of \( h \) when \( b = 1, 3, 6, 9, \) and 12.

\( d. \) Numerical Write an inequality for the possible lengths of \( b \) such that \( b < h \). Write an inequality for the possible lengths of \( h \) such that \( b > h \).

**H.O.T. Problems** Use Higher-Order Thinking Skills

42. ERROR ANALYSIS Taro and Jamie are solving \( 6x \geq -84 \). Is either of them correct? Explain your reasoning.

\[ \begin{array}{|c|c|}
\hline
\text{Taro} & \text{Jamie} \\
\hline
4x \geq -6 & 6x \geq -48 \\
2x \geq -14 & x \geq -4 \\
\hline
\end{array} \]

43. CHALLENGE Solve each inequality for \( x \). Assume that \( a > 0 \).

\( a. \) \( -ax < b \)
\( b. \) \( \frac{2}{3}x \geq 8 \)
\( c. \) \( -6 \geq ax \)

44. CSS STRUCTURE Determine whether \( x > 1 \) and \( x > 1 \) are equivalent. Explain.

45. REASONING Explain whether the statement \( \frac{a}{b} > 1 \) is sometimes, always, or never true.

46. OPEN ENDED Create a real-world situation to represent the inequality \( \frac{3}{8} < x \).

47. WRITING IN MATH How are solving linear inequalities and linear equations similar? different?

**Spiral Review**

Solve each inequality. Check your solution, and then graph it on a number line.

52. \( -8 + 4a < 6a \)
53. \( 2y + 11 \geq 2y - 3 \)
54. \( 7 - 2b > 12b \)

Find the inverse of each function.

55. \( f(x) = -6x + 18 \)
56. \( f(x) = \frac{3}{2}x + 9 \)
57. \( f(x) = 4x - 5 \)

58. HOME DECOR Pam is having blinds installed at her home. The cost \( c \) of installation for any number of blinds \( b \) can be described by \( c = 25 + 6.5b \). Graph the equation and determine how much it would cost if Pam has 8 blinds installed.

59. RESCUE A boat required for a helicopter to pick up a sick crew member. At that time, the boat and the helicopter were at the positions shown. How long will it take for the helicopter to reach the boat?

Solve each equation.

60. \( |x + 3| = 10 \)
61. \( |2x - 8| = 6 \)
62. \( |3x + 1| = -2 \)

**Skills Review**

Solve each equation.

63. \( 4x + 11 = 19 \)
64. \( 2x - 7 = 9 + 4x \)
65. \( \frac{1}{4} + 2x = 4x - 8 \)
66. \( \frac{1}{3}(6t - 3) = 3t + 12 \)
67. \( \frac{7t + 5}{2} = 13 \)
68. \( \frac{1}{2}x = x - 3 \frac{3}{4} \)