Let $f: \{0,1\}^n \rightarrow \{0,1\}$ be a function.

**Def. (Deterministic query complexity)** $D(f)$ is the number of queries to $x$ needed to compute $f(x)$.

**Def. (Quantum query complexity)** $Q(f)$ is the number of quantum queries to $x$ needed to compute $f(x)$.

**Def. (Sensitivity)** $s(f) = \max \{ \{ i \mid \text{ bit flipped} \} : x, y : (x_i = 1) \land (y_i = 0) \}$

**Corollary of sensitivity theorem**

- $D(f) = O(s(f)^2)$
- $Q(f) = O(s(f)^4)$
- $D(f) = O(CYQ)^{1/3}$ — limit to how much better quantum algorithms can be.

**Sensitivity Theorem**: If $G = (V, E)$ is the hypercube $G = (\{0,1\}^n, E)$, an induced subgraph, $|V| \geq 2^n$, largest degree in $G$ is at least $\Theta(1/\sqrt{n})$.

Theorem: $g: \{0,1\}^n \rightarrow \{0,1\}$ is a function.

- $g(x) = 0$ if $x$ has an odd # of $1$s.
- $g(x) = 1$ if $x$ has an even # of $1$s.

$g$ is a generator on $\text{el}$.

Let $f: \{0,1\}^n \rightarrow \{0,1\}$.

If $\sum_{x \in \{0,1\}^n} g_x \cdot f(x) = 0$, $s(f) \geq \delta n$

Proof. Let $V' = \{ x \mid x \in \{0,1\}^n, \sum_{x \in \{0,1\}^n} g_x \cdot f(x) = 0 \}$.
\[ 1 \leq (\frac{1}{2}) \cdot g_u + \epsilon \quad s(k) \geq 5n \]

**Proof** Let \( V' = \{ v \mid v \in \{0,1\}^n, (\cdot)^e = g_v \} \)

Either \( |V'| < 2^{-n} \) or \( |V'| > 2^{-n} \)

(If \( |V'| = 2^{-n}, \sum_{v \in \{0,1\}^n} g_v = 0 \))

Assume \( |V'| > 2^{-n} \) (otherwise \( V \equiv V' \))

By (\( \ast \)), there exist \( v \in V' \) s.t.

Then if \( \sum_{v \in \{0,1\}^n} g_v \neq 0 \),

\[ D_c(f) \leq n^2 \]